AP MC - Unit Review

① $y = x\cos x$ PRODUCT RULE!

$y' = \overset{F}{(x)}\overset{S'}{(-\sin x)} + \overset{S}{(\cos x)}\overset{F'}{(1)} = 0$

$= -x\sin x + \cos x = 0$

$\dfrac{-x\cancel{\sin x}}{\cancel{\sin x}} = \dfrac{-\cos x}{\sin x}$

$-x = -\cot x$

$x = \cot x$

$\dfrac{1}{x} = \dfrac{1}{\cot x}$

B $\dfrac{1}{x} = \tan x$

have to keep manipulating the problem until you get an answer that is listed

② $\cos x = e^y$

$\dfrac{d}{dx}(\cos x = e^y) =$

$-\sin x = e^y \dfrac{dy}{dx}$

$\dfrac{dy}{dx} = \dfrac{-\sin x}{e^y}$

But answer must be all in terms of x

$\dfrac{dy}{dx} = \dfrac{-\sin x}{e^y}$ $e^y = \cos x$ substitute original problem!

$\dfrac{dy}{dx} = \dfrac{-\sin x}{\cos x} = -\tan x$ A

③ If $f(x) = g[x + g(x)]$ chain rule!

then $f'(x) = g'[x + g(x)] \cdot (1 + g'(x))$

take derivative of f(x) and THEN plug in $x=1$

$f'(1) = g'[1 + g(1)] \cdot (1 + g'(1))$

$= g'[1 + 3] \cdot (1 + -2)$

$= g'[4] \cdot (-1)$

$= \frac{2}{3} \cdot -1 = \frac{-2}{3}$ Ⓔ

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④ $y = \cos^2 x - \sin^2 x$ $\xrightarrow{\text{FIRST REWRITE:}}$ $y = (\cos x)^2 - (\sin x)^2$

SECOND, CHAIN RULE!

$y' = 2(\cos x) \cdot -\sin x - 2(\sin x)\cos x$

$= -2\cos x \sin x - 2\cos x \sin x$

$= -4\cos x \sin x$

$= -4(\cos x)(\sin x)$ Ⓔ

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⑤ [Review: odd function = $f(-x) = -f(x)$
even function = $f(-x) = f(x)$]

The problem told us "$g(-x) = g(x)$" which means that it is an even function.

Choose any random even function and see what happens when you find $g'(a)$.

even function!

$g(x) = x^2$

$g'(x) = 2x$

$g'(-a) = 2(-a) = -2a$

So for this even function, $g'(-a) = -g'(a)$ Ⓑ

⑥ $y = \arctan \frac{x}{3}$

need point: $\overset{\text{given}}{(0,0)}$ and slope: ?

slope $= y' = \frac{1}{1+(\frac{x}{3})^2} \cdot \frac{1}{3}$

$y'(0) = \frac{1}{1+(\frac{0}{3})^2} \cdot \frac{1}{3} = \frac{1}{1+0} \cdot \frac{1}{3} = \frac{1}{3}$ } slope

$y - y_0 = m(x - x_0)$

$y - 0 = \frac{1}{3}(x - 0)$

$y = \frac{1}{3}x$

$3(y = \frac{1}{3}x) = 3y = x$

$\quad -3y \quad -3y$

Just keep manipulating the problem until you get an answer that is listed →

$0 = x - 3y$ (A)

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⑦ $\frac{d}{dx}(\ln e^{3x}) =$

can do 2 different ways:

[I think the first way is the best]

1st way:

rewrite equation $y = \ln e^{3x}$.
ln + e are inverses so this problem is $y = 3x$

Now take derivative.

$y' = 3$ (B)

2nd way:

take derivative from beginning

$y = \ln e^{3x}$

$y' = \frac{1}{e^{3x}} \cdot e^{3x} \cdot \cancel{\ln e}^{1} \cdot 3$

$= \frac{\cancel{e^{3x}} \cdot 3}{\cancel{e^{3x}}} = 3$ (B)

⑧ $y^2 - 3x = 7$ Find $\frac{d^2y}{dx^2}$

$$\frac{d}{dx}\left(y^2 - 3x = 7\right) = 2y\frac{dy}{dx} - 3 = 0$$

$$2y\frac{dy}{dx} = 3$$

$$\frac{dy}{dx} = \frac{3}{2y}$$ ← FIRST DERIVATIVE ANSWER

$$\frac{d}{dx}\left(\frac{dy}{dx} = \frac{3}{2y}\right) = \frac{d^2y}{dx^2} = \frac{\cancel{(2y)(0)} - (3)(2\frac{dy}{dx})}{(2y)^2}$$

$$= \frac{-3 \cdot 2 \cdot \frac{3}{2y}}{4y^2} = \frac{\frac{-18}{2y}}{4y^2}$$

$$= \frac{(-18)}{(2y)(4y^2)} = \frac{-18}{8y^3} = \frac{-9}{4y^3}$$ (E)

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⑨ $h(x) = (x^2 - 4)^{3/4} + 1$ Find $h'(2)$

$$h'(x) = \frac{3}{4}(x^2-4)^{-1/4} \cdot 2x = \frac{3}{4} \cdot \frac{2x}{(x^2-4)^{1/4}} = \frac{3(2x)}{4(x^2-4)^{1/4}}$$

$$h'(2) = \frac{6(2)}{4(2^2-4)^{1/4}} = \frac{12}{4(0)} = \frac{3}{0} = \text{DNE}$$ (E)

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⑩ $$\lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h} =$$

this is set up as the definition of the derivative
so this is asking what is the derivative of:

$y = \cos x$

$y' = -\sin x$ (B)

(11) $p(x) = (x-1)(x+k)$ is parallel to $5x - y + 6 = 0$

rewrite the above eq.

$-y = -6 - 5x$

$y = 5x + 6$

Since $p(x)$ is parallel to this eq. they have the same slope.... 5

Find the slope of $p(x)$ and set equal to 5.

product rule!

$$p'(x) = \overset{F}{(x-1)}\overset{S'}{(1)} + \overset{S}{(x+k)}\overset{F'}{(1)} = 5$$

$$x - 1 + x + k = 5$$

$$p'(4) = (4) - 1 + (4) + k = 5$$

$$8 - 1 + k = 5$$

$$7 + k = 5$$

$$k = -2 \quad \text{E}$$

(12) $\tan(x+y) = x$

distribute

$$\frac{dy}{dx}\left(\tan(x+y) = x\right) = \sec^2(x+y) \cdot \left(1 + \frac{dy}{dx}\right) = 1$$

$$= \sec^2(x+y) + \sec^2(x+y)\frac{dy}{dx} = 1$$

$$\sec^2(x+y)\frac{dy}{dx} = 1 - \sec^2(x+y)$$

$$\frac{dy}{dx} = \frac{1 - \sec^2(x+y)}{\sec^2(x+y)}$$

$$\frac{dy}{dx} = \frac{1}{\sec^2(x+y)} - \frac{\cancel{\sec^2(x+y)}}{\cancel{\sec^2(x+y)}}\ 1$$

$$\frac{dy}{dx} = \cos^2(x+y) - 1 \quad \text{E}$$

(13) SKIP

$y = \frac{\ln e^{2x}}{x-1} = \frac{2x}{x-1}$

quotient rule $y' = \frac{(\overset{B}{x-1})(\overset{T'}{2}) - (\overset{T}{2x})(\overset{B'}{\ })}{(x-1)^2}$

$y' = \frac{\cancel{2x} - 2 - \cancel{2x}}{(x-1)^2} = \frac{-2}{(x-1)^2}$

$\rightarrow$ inverse $= \frac{1}{y'} = \frac{1}{\left(\frac{-2}{(x-1)^2}\right)}$

$= \frac{(x-1)^2}{-2}$

at $x=3$:

$\frac{(3-1)^2}{-2} = -2$ (E)

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(14) $y = f(g(x))$

CHAIN RULE

$y' = f'(g(x)) \cdot g'(x)$

$y'(e) = f'(g(e)) \cdot g'(e)$

$= f'(1) \cdot \frac{1}{e}$

$= 2e^2 \cdot \frac{1}{e} = \frac{2e^2}{e} = 2e$ (B) AND (C)

$f(x) = e^{2x}$

$f'(x) = 2e^{2x}$

$f'(e) = 2e^{2e}$

$f'(1) = 2e^2$

$g(x) = \ln x$

$g'(x) = \frac{1}{x}$

$g'(e) = \frac{1}{e}$

$g(e) = \ln e = 1$

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(15) $y = \sin u$ $\quad u = v - \frac{1}{v}$ $\quad v = \ln x$

Rewrite as $y = \sin\left(\ln x - \frac{1}{\ln x}\right)$

THEN take derivative and plug in $x = e$

$y' = \cos\left(\ln x - \frac{1}{\ln x}\right) \cdot \left(\frac{1}{x} - \frac{\cancel{(\ln x)(0)} - (1)(\frac{1}{x})}{(\ln x)^2}\right)$

$= \cos\left(\ln x - \frac{1}{\ln x}\right) \cdot \left(\frac{1}{x} + \frac{1}{x(\ln x)^2}\right)$

$y'(e) = \cos\left(\ln e - \frac{1}{\ln e}\right) \cdot \left(\frac{1}{e} + \frac{1}{e(\ln e)^2}\right)$

$= \cos(1-1) \cdot \left(\frac{1}{e} + \frac{1}{e \cdot 1^2}\right)$

$= \cos 0 \cdot \left(\frac{2}{e}\right) = (1) \cdot \left(\frac{2}{e}\right) = \frac{2}{e}$ (D)

(16) $x + xy + 2y^2 = 6$

$\frac{d}{dx}\left(x + xy + 2y^2 = 6\right) = 1 + x\frac{dy}{dx} + y + 4y\frac{dy}{dx} = 0$

implicit differentiation

$x\frac{dy}{dx} + 4y\frac{dy}{dx} = -1 - y$

$\frac{dy}{dx}(x + 4y) = -1 - y$

$\frac{dy}{dx} = \frac{-1-y}{x+4y}$

Then plug in given pt (2,1)

$\frac{dy}{dx} = \frac{-1-(1)}{(2)+4(1)} = \frac{-2}{6} = -\frac{1}{3}$ (C)

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(17) $\lim_{h\to 0} \frac{g(x+h) - g(x)}{h}$ This is in the set up of the definition of the derivative.

Just find the derivative of $g(x)$

$y = g(x) = x + \cos x$

$y' = g'(x) = 1 + -\sin x$

$= 1 - \sin x$ (C)

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(18) $f(x) = \sin^2 x$ $g(x) = 0.5x^2$

on calculator:

$\frac{d}{dx}(f(x))\Big|_{x=-.8, 0, .9...}$ go through each mult. choice answer and find derivative value. See when f(x) is greater than g(x)

$\frac{d}{dx}(g(x))\Big|_{x=-.8, 0, ...9}$ answer (C)

(20) $e^y = xy$

$$\frac{d}{dx}\left(e^y = xy\right) = \quad e^y \frac{dy}{dx} = x\frac{dy}{dx} + y$$

implicit differentiation

$$e^y \frac{dy}{dx} - x\frac{dy}{dx} = y$$

$$\frac{dy}{dx}\left(e^y - x\right) = y$$

$$\frac{dy}{dx} = \frac{y}{e^y - x} = \frac{y}{xy - x} \quad \text{(E)}$$

Sub: $e^y = xy$

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(21) $(x-y)^2 = y^2 - xy$ *Must be really good at algebra to solve correctly!

$$\frac{d}{dx}\left((x-y)^2 = y^2 - xy\right) = \quad 2(x-y)\cdot\left(1-\frac{dy}{dx}\right) = 2y\frac{dy}{dx} - \left(x\frac{dy}{dx} + y\right)$$

distribute   distribute

implicit differentiation

FOIL

$$(2x-2y)\left(1-\frac{dy}{dx}\right) = 2y\frac{dy}{dx} - x\frac{dy}{dx} - y$$

$$2x - 2x\frac{dy}{dx} - 2y + \cancel{2y\frac{dy}{dx}} = \cancel{2y\frac{dy}{dx}} - x\frac{dy}{dx} - y$$

$$-2x\frac{dy}{dx} + x\frac{dy}{dx} = -2x + 2y - y$$

$$-x\frac{dy}{dx} = -2x + y$$

$$\frac{dy}{dx} = \frac{-2x + y}{-x} = \frac{\cancel{-1}(2x-y)}{\cancel{-1}(x)}$$

$$= \frac{2x-y}{x} \quad \text{(C)}$$

(22) given: $g(a+b) - g(a) = 4ab + 2b^2$

notice this form is related to $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$= \lim_{b\to 0} \frac{(a+b) - g(a)}{b} = \lim_{b \to 0} \frac{4ab + 2b^2}{b}$$

$$= \lim_{b\to 0} \frac{4a\cancel{b}}{\cancel{b}} + \frac{2b^{\cancel{2}}}{\cancel{b}} = \lim_{b\to 0} 4a + 2b = 4a + 2(0)$$

$= 4a$ { We took the derivative w.r.t. a
If we took the derivative w.r.t. x....

$= 4x$ (D)

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(23) $\frac{d}{dx}(\arctan 4x) = \frac{1}{1+(4x)^2} \cdot 4 = \frac{4}{1+16x^2}$ (B)

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(24) $g(x) = \sqrt[3]{x-1} \longrightarrow$

$y = \sqrt[3]{x-1}$

$y = (x-1)^{1/3}$

find inverse:
(swith x + y)

$x = (y-1)^{1/3}$

Solve for y

$x^3 = y-1$

$y = x^3 + 1$

Differentiate!

$y' = 3x^2$ (A)

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(25) $y = u + 2e^u$ + $u = 1 + \ln x$

rewrite: $y = 1 + \ln x + 2e^{1+\ln x}$

$$\frac{dy}{dx} = \cancel{0} + \frac{1}{x} + 2(e^{1+\ln x} \cdot \cancel{\ln e} \cdot \tfrac{1}{x})$$

$$= \frac{1}{x} + 2e^{1+\ln x} \cdot \frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=\frac{1}{e}} = \frac{1}{(1/e)} + 2e^{1+\cancel{\ln e}^{-1}} \cdot \frac{1}{1/e}$$

$$= e + 2e^{1+-1} \cdot e$$

$$= e + 2e^{0} \cdot e$$

$$= e + 2e = 3e$$ (C)