Question 2

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

(a) $\int_9^{17} E(t)\,dt = 6004.270$ 6004 people entered the park by 5 pm.	$3 \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $15\int_9^{17} E(t)\,dt + 11\int_{17}^{23} E(t)\,dt = 104048.165$ The amount collected was \$104,048. or $\int_{17}^{23} E(t)\,dt = 1271.283$ 1271 people entered the park between 5 pm and	$1: \text{setup}$
11 pm, so the amount collected was $\$15 \cdot (6004) + \$11 \cdot (1271) = \$104,041.$	
(c) $H'(17) = E(17) - L(17) = -380.281$ There were 3725 people in the park at $t = 17$. The number of people in the park was decreasing at the rate of approximately 380 people/hr at time $t = 17$.	$3 \begin{cases} 1: \text{value of } H'(17) \\ 2: \text{meanings} \\ \quad 1: \text{meaning of } H(17) \\ \quad 1: \text{meaning of } H'(17) \\ \quad <-1> \text{ if no reference to } t = 17 \end{cases}$
(d) $H'(t) = E(t) - L(t) = 0$ $t = 15.794 \text{ or } 15.795$	$2 \begin{cases} 1: E(t) - L(t) = 0 \\ 1: \text{answer} \end{cases}$

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Question 2

The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

(a) $P'(9) = 1 - 3e^{-0.6} = -0.646 < 0$
so the amount is not increasing at this time.

| | 1 : answer with reason |

(b) $P'(t) = 1 - 3e^{-0.2\sqrt{t}} = 0$
$t = (5\ln 3)^2 = 30.174$
$P'(t)$ is negative for $0 < t < (5\ln 3)^2$ and positive for $t > (5\ln 3)^2$. Therefore there is a minimum at $t = (5\ln 3)^2$.

| | 3 : $\begin{cases} 1 : \text{sets } P'(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{justification} \end{cases}$ |

(c) $P(30.174) = 50 + \int_0^{30.174} \left(1 - 3e^{-0.2\sqrt{t}}\right) dt$
  $= 35.104 < 40$, so the lake is safe.

| | 3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{conclusion with reason} \\ \quad \text{based on integral of } P'(t) \end{cases}$ |

(d) $P'(0) = 1 - 3 = -2$. The lake will become safe when the amount decreases by 10. A linear model predicts this will happen when $t = 5$.

| | 2 : $\begin{cases} 1 : \text{slope of tangent line} \\ 1 : \text{answer} \end{cases}$ |

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Question 2

A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t+1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12\sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

(a) $\displaystyle\int_0^{12} H(t)\,dt = 70.570$ or 70.571

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $H(6) - R(6) = -2.924,$
so the level of heating oil is falling at $t = 6$.

$1 : \text{answer with reason}$

(c) $125 + \displaystyle\int_0^{12} (H(t) - R(t))\,dt = 122.025$ or 122.026

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(d) The absolute minimum occurs at a critical point or an endpoint.

$H(t) - R(t) = 0$ when $t = 4.790$ and $t = 11.318$.

The volume increases until $t = 4.790$, then decreases until $t = 11.318$, then increases, so the absolute minimum will be at $t = 0$ or at $t = 11.318$.

$125 + \displaystyle\int_0^{11.318} (H(t) - R(t))\,dt = 120.738$

Since the volume is 125 at $t = 0$, the volume is least at $t = 11.318$.

$3 : \begin{cases} 1 : \text{sets } H(t) - R(t) = 0 \\ 1 : \text{volume is least at} \\ \quad t = 11.318 \\ 1 : \text{analysis for absolute} \\ \quad \text{minimum} \end{cases}$

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Question 2

A water tank at Camp Newton holds 1200 gallons of water at time $t = 0$. During the time interval $0 \leq t \leq 18$ hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275 \sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

(a) Is the amount of water in the tank increasing at time $t = 15$? Why or why not?

(b) To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

(c) At what time t, for $0 \leq t \leq 18$, is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.

(d) For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Let k be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

(a) No; the amount of water is not increasing at $t = 15$ since $W(15) - R(15) = -121.09 < 0$.

1 : answer with reason

(b) $1200 + \int_0^{18} (W(t) - R(t))\, dt = 1309.788$

1310 gallons

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $W(t) - R(t) = 0$
$t = 0, 6.4948, 12.9748$

t (hours)	gallons of water
0	1200
6.495	525
12.975	1697
18	1310

The values at the endpoints and the critical points show that the absolute minimum occurs when $t = 6.494$ or 6.495.

$3 : \begin{cases} 1 : \text{interior critical points} \\ 1 : \text{amount of water is least at} \\ \quad t = 6.494 \text{ or } 6.495 \\ 1 : \text{analysis for absolute minimum} \end{cases}$

(d) $\int_{18}^{k} R(t)\, dt = 1310$

$2 : \begin{cases} 1 : \text{limits} \\ 1 : \text{equation} \end{cases}$

Question 2

The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by

$$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S, given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

(a) How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.

(b) Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t.

(c) Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.

(d) For $0 \le t \le 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.

(a) $\displaystyle\int_0^6 R(t)\,dt = 31.815$ or 31.816 yd^3

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer with units} \end{cases}$

(b) $Y(t) = 2500 + \displaystyle\int_0^t (S(x) - R(x))\,dx$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(c) $Y'(t) = S(t) - R(t)$

$Y'(4) = S(4) - R(4) = -1.908$ or -1.909 yd^3/hr

$1 : \text{answer}$

(d) $Y'(t) = 0$ when $S(t) - R(t) = 0$.
The only value in $[0, 6]$ to satisfy $S(t) = R(t)$
is $a = 5.117865$.

$3 : \begin{cases} 1 : \text{sets } Y'(t) = 0 \\ 1 : \text{critical } t\text{-value} \\ 1 : \text{answer with justification} \end{cases}$

t	$Y(t)$
0	2500
a	2492.3694
6	2493.2766

The amount of sand is a minimum when $t = 5.117$ or 5.118 hours. The minimum value is 2492.369 cubic yards.

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Question 1

Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function F defined by

$$F(t) = 82 + 4\sin\left(\frac{t}{2}\right) \text{ for } 0 \le t \le 30,$$

where $F(t)$ is measured in cars per minute and t is measured in minutes.

(a) To the nearest whole number, how many cars pass through the intersection over the 30-minute period?

(b) Is the traffic flow increasing or decreasing at $t = 7$? Give a reason for your answer.

(c) What is the average value of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(d) What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

(a) $\int_0^{30} F(t)\, dt = 2474$ cars

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(b) $F'(7) = -1.872$ or -1.873
Since $F'(7) < 0$, the traffic flow is decreasing at $t = 7$.

$1 :$ answer with reason

(c) $\frac{1}{5}\int_{10}^{15} F(t)\, dt = 81.899$ cars/min

$$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$$

(d) $\dfrac{F(15) - F(10)}{15 - 10} = 1.517$ or 1.518 cars$/\text{min}^2$

$1 :$ answer

Units of cars/min in (c) and cars/min.² in (d)

$1 :$ units in (c) and (d)

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \leq t \leq 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

(a) How many people are in the auditorium when the concert begins?

(b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

(d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a) $\int_0^2 R(t)\,dt = 980$ people

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$

The maximum rate may occur at 0, $a = 1.36296$, or 2.

$R(0) = 0$
$R(a) = 854.527$
$R(2) = 120$

The maximum rate occurs when $t = 1.362$ or 1.363.

$3 : \begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^2 w'(t)\,dt = \int_1^2 (2 - t)R(t)\,dt = 387.5$

The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{1}{980}w(2) = \frac{1}{980}\int_0^2 (2 - t)R(t)\,dt = 0.77551$
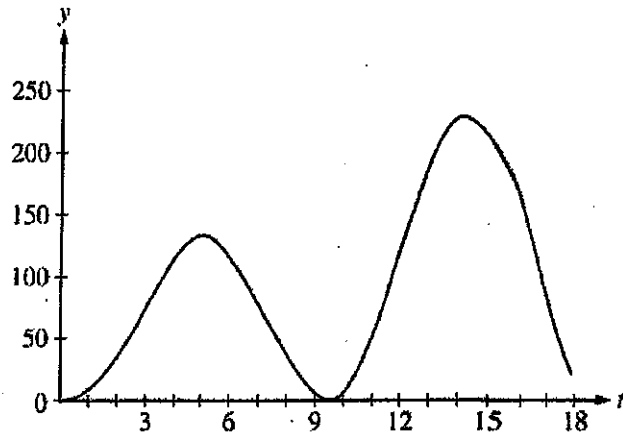
On average, a person waits 0.775 or 0.776 hour.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

Question 2

At an intersection in Thomasville, Oregon, cars turn left at the rate $L(t) = 60\sqrt{t} \sin^2\left(\dfrac{t}{3}\right)$ cars per hour over the time interval $0 \le t \le 18$ hours. The graph of $y = L(t)$ is shown above.

(a) To the nearest whole number, find the total number of cars turning left at the intersection over the time interval $0 \le t \le 18$ hours.

(b) Traffic engineers will consider turn restrictions when $L(t) \ge 150$ cars per hour. Find all values of t for which $L(t) \ge 150$ and compute the average value of L over this time interval. Indicate units of measure.

(c) Traffic engineers will install a signal if there is any two-hour time interval during which the product of the total number of cars turning left and the total number of oncoming cars traveling straight through the intersection is greater than 200,000. In every two-hour time interval, 500 oncoming cars travel straight through the intersection. Does this intersection require a traffic signal? Explain the reasoning that leads to your conclusion.

(a) $\displaystyle\int_0^{18} L(t)\,dt \approx 1658$ cars

$2 : \begin{cases} 1 : \text{setup} \\ 1 : \text{answer} \end{cases}$

(b) $L(t) = 150$ when $t = 12.42831,\ 16.12166$
Let $R = 12.42831$ and $S = 16.12166$
$L(t) \ge 150$ for t in the interval $[R, S]$

$\dfrac{1}{S - R}\displaystyle\int_R^S L(t)\,dt = 199.426$ cars per hour

$3 : \begin{cases} 1 : t\text{-interval when } L(t) \ge 150 \\ 1 : \text{average value integral} \\ 1 : \text{answer with units} \end{cases}$

(c) For the product to exceed 200,000, the number of cars turning left in a two-hour interval must be greater than 400.

$\displaystyle\int_{13}^{15} L(t)\,dt = 431.931 > 400$

$4 : \begin{cases} 1 : \text{considers 400 cars} \\ 1 : \text{valid interval } [h,\, h+2] \\ 1 : \text{value of } \int_h^{h+2} L(t)\,dt \\ 1 : \text{answer and explanation} \end{cases}$

OR

The number of cars turning left will be greater than 400 on a two-hour interval if $L(t) \ge 200$ on that interval. $L(t) \ge 200$ on any two-hour subinterval of $[13.25304, 15.32386]$.

OR

$4 : \begin{cases} 1 : \text{considers 200 cars per hour} \\ 1 : \text{solves } L(t) \ge 200 \\ 1 : \text{discusses 2 hour interval} \\ 1 : \text{answer and explanation} \end{cases}$

Yes, a traffic signal is required.

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Question 1

At a certain height, a tree trunk has a circular cross section. The radius $R(t)$ of that cross section grows at a rate modeled by the function

$$\frac{dR}{dt} = \frac{1}{16}\left(3 + \sin\left(t^2\right)\right) \text{ centimeters per year}$$

for $0 \leq t \leq 3$, where time t is measured in years. At time $t = 0$, the radius is 6 centimeters. The area of the cross section at time t is denoted by $A(t)$.

(a) Write an expression, involving an integral, for the radius $R(t)$ for $0 \leq t \leq 3$. Use your expression to find $R(3)$.

(b) Find the rate at which the cross-sectional area $A(t)$ is increasing at time $t = 3$ years. Indicate units of measure.

(c) Evaluate $\int_0^3 A'(t)\, dt$. Using appropriate units, interpret the meaning of that integral in terms of cross-sectional area.

(a) $R(t) = 6 + \int_0^t \frac{1}{16}\left(3 + \sin\left(x^2\right)\right) dx$

$R(3) = 6.610$ or 6.611

$3 : \begin{cases} 1 : \text{integral} \\ 1 : \text{expression for } R(t) \\ 1 : R(3) \end{cases}$

(b) $A(t) = \pi (R(t))^2$

$A'(t) = 2\pi R(t) R'(t)$

$A'(3) = 8.858 \text{ cm}^2/\text{year}$

$3 : \begin{cases} 1 : \text{expression for } A(t) \\ 1 : \text{expression for } A'(t) \\ 1 : \text{answer with units} \end{cases}$

(c) $\int_0^3 A'(t)\, dt = A(3) - A(0) = 24.200$ or 24.201

From time $t = 0$ to $t = 3$ years, the cross-sectional area grows by 24.201 square centimeters.

$3 : \begin{cases} 1 : \text{uses Fundamental Theorem of Calculus} \\ 1 : \text{value of } \int_0^3 A'(t)\, dt \\ 1 : \text{meaning of } \int_0^3 A'(t)\, dt \end{cases}$

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Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t)\, dt = 171.813$ mm

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\dfrac{S(60) - S(0)}{60} = 2.863$ or 2.864 mm/day

$1 : \text{answer}$

(c) $V(t) = 100\pi S(t)$

$V'(7) = 100\pi S'(7) = 602.218$

The volume of water in the can is increasing at a rate of 602.218 mm³/day.

$2 : \begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t, $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

$2 : \begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

$1 : \text{units in (b) or (c)}$

Question 3

The wind chill is the temperature, in degrees Fahrenheit (°F), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v, in miles per hour (mph). If the air temperature is 32°F, then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \le v \le 60$.

(a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.

(b) Find the average rate of change of W over the interval $5 \le v \le 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \le v \le 60$.

(c) Over the time interval $0 \le t \le 4$ hours, the air temperature is a constant 32°F. At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at 0.286 °F/mph.

$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$

(b) The average rate of change of W over the interval
$5 \le v \le 60$ is $\dfrac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254.

$W'(v) = \dfrac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

$3 : \begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$

(c) $\dfrac{dW}{dt}\bigg|_{t=3} = \left(\dfrac{dW}{dv} \cdot \dfrac{dv}{dt}\right)\bigg|_{t=3} = W'(35) \cdot 5 = -0.892$ °F/hr

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\dfrac{dW}{dt}\bigg|_{t=3} = -0.892$ °F/hr

Units of °F/mph in (a) and °F/hr in (c)

$3 : \begin{cases} 1 : \dfrac{dv}{dt} = 5 \\ 1 : \text{uses } v(3) = 35, \\ \quad \text{or} \\ \quad \text{uses } v(t) = 20 + 5t \\ 1 : \text{answer} \end{cases}$

$1 :$ units in (a) and (c)

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