Name_____

Separable Differential Equations

Date_____ Period____

Find the general solution of each differential equation.

1) $\frac{dy}{dx} = e^{x-y}$

$dy = e^x \cdot e^{-y}\, dx$

$\frac{1}{e^{-y}}\, dy = e^x\, dx$

$\int e^y\, dy = \int e^x\, dx$

$\ln(e^y) = \ln(e^x + c)$

$\boxed{y = \ln(e^x + c)}$

2) $\frac{dy}{dx} = \frac{1}{\sec^2 y}$

$dy = \frac{1}{\sec^2 y}\, dx$

$\int \sec^2 y\, dy = \int dx$

$\arctan(\tan y) = (x + c)$

$\boxed{y = \arctan(x + c)}$

3) $\frac{dy}{dx} = xe^y$

$\frac{dy}{e^y} = x\, dx$

$\int e^{-y}\, dy = \int x\, dx$

$-e^{-y} = \frac{1}{2}x^2 + c$

$\ln e^{-y} = \ln\left(-\frac{1}{2}x^2 + c\right)$

$-y = \ln\left(-\frac{1}{2}x^2 + c\right)$

$\boxed{y = -\ln\left(-\frac{1}{2}x^2 + c\right)}$

4) $\frac{dy}{dx} = \frac{2x}{e^{2y}}$

$\int e^{2y}\, dy = \int 2x\, dx$

$\frac{1}{2}e^{2y} = x^2 + c$

$e^{2y} = 2(x^2 + c)$

$\ln e^{2y} = \ln 2x^2 + c$

$2y = \ln(2x^2 + c)$

$\boxed{y = \frac{1}{2}\ln(2x^2 + c)}$

5) $\frac{dy}{dx} = 2y - 1$

$\int \frac{1}{2y-1}\, dy = \int dx$

$\frac{1}{2}\ln|2y-1| = x + c$

$e^{\ln|2y-1|} = e^{2x+c}$

$2y - 1 = e^{2x+c}$

$2y = e^{2x+c} + 1$

$y = \frac{1}{2}(e^{2x+c} + 1)$

$\boxed{y = \frac{1}{2}(ce^{2x} + 1)}$

6) $\frac{dy}{dx} = 2yx + yx^2$

$dy = y(2x + x^2)\, dx$

$\int \frac{1}{y}\, dy = \int 2x + x^2\, dx$

$\ln|y| = x^2 + \frac{1}{3}x^3 + c$

$e^{\ln|y|} = e^{x^2 + \frac{1}{3}x^3 + c}$

$y = e^{x^2 + \frac{1}{3}x^3 + c}$

$y = e^{x^2 + \frac{1}{3}x^3} \cdot e^c$

$\boxed{y = ce^{x^2 + \frac{1}{3}x^3}}$

1. $\int y^2\,dy = \int (1-x)\,dx$

$$\frac{1}{3}y^3 = x - \frac{1}{2}x^2 + c$$

$$y^3 = 3x - \frac{3}{2}x^2 + 3c$$

$$y = \left(3x - \frac{3}{2}x^2 + \underbrace{3c}_{c}\right)^{1/3}$$

2. $y'\sin(2y) - \cos x = 0, \qquad x = \pi/2,\ y = 0$

$$\frac{dy}{dx}\sin 2y = \cos x$$

$$\int dy \sin 2y = \int \cos x\,dx$$

$$-\frac{1}{2}\cos 2y = \sin x + c$$

$$-\frac{1}{2}\cos 2(0) = \sin(\pi/2) + c$$

$$-\frac{1}{2}\cdot 1 = 1 + c$$

$$c = -3/2$$

$$-\frac{1}{2}\cos 2y = \sin x - \frac{3}{2}$$

$$\arccos(\cos 2y) = (-2\sin x + 3)$$

$\arccos$

$$2y = \arccos(-2\sin x + 3)$$

$$y = \frac{1}{2}\arccos(-2\sin x + 3)$$