U9H2

Determine whether or not the function y satisfies the differential equation.

1. $y' = y$ $\quad y = e^x$ $\quad y' = e^x$

$e^x = e^x$

✓ yes

2. $\dfrac{dy}{dx} = 2xy$ $\quad y = e^{x^2} + 5$ $\quad y' = 2xe^{x^2}$

$2xe^{x^2} = 2x(e^{x^2} + 5)$

$2xe^{x^2} = 2xe^{x^2} + 10x$

X no

Find the general solution to the exact differential equation.

3. $\dfrac{dy}{dx} = \sec x \tan x - e^x$

$\int dy = \int \sec x \tan x - e^x \, dx$

$y + c_1 = \sec x - e^x + c_2$

$y = \sec x - e^x + c$

typical (general) solution

4. $\dfrac{dy}{dt} = 3t^2 \cos(t^3)$

chain rule!

$\int dy = \int 3t^2 \cos t^3$

$y + c_1 = \sin t^3 + c_2$

$y = \sin t^3 + c$

typical solution (general)

Solve the initial value problem explicitly.

5. $\dfrac{dy}{dx} = 2e^x - \cos x \qquad y(0) = 3$

$\int dy = \int 2e^x - \cos x \, dx$

$y + c_1 = 2e^x - \sin x + c_2$

typical solution $y = 2e^x - \sin x + c$

$3 = 2e^0 - \sin 0 + c$

$3 = 2 - (0) + c$

$3 = 2 + c$

$c = 1$

exact solution $y = 2e^x - \sin x + 1$

6. $s'(t) = t(3t - 2) \qquad s(1) = 0$

$\dfrac{ds}{dt} = 3t^2 - 2t$

$\int ds = \int 3t^2 - 2t \, dt$

$s + c_1 = t^3 - t^2 + c_2$

$s = t^3 - t^2 + c$ typical solution

$0 = 1^3 - 1^2 + c$

$c = 0$

$s = t^3 - t^2$ exact solution

7. $\dfrac{y+2}{x^2 - x + 2} \cdot \dfrac{dy}{dx} = \dfrac{x}{y} \qquad y(1) = 2$

$y(y+2)\, dy = x(x^2 - x + 2)\, dx$

$\int y^2 + 2y \, dy = \int x^3 - x^2 + 2x \, dx$

$\dfrac{y^3}{3} + y^2 + c_1 = \dfrac{x^4}{4} - \dfrac{x^3}{3} + x^2 + c_2$

typical solution $y^3 + y^2 = \frac{1}{4}x^4 - \frac{1}{3}x^3 + x^2 + c$

$2^3 + 2^2 = \frac{1}{4}(1)^4 - \frac{1}{3}(1)^3 + (1)^2 + c$

$8 + 4 = \frac{1}{4} - \frac{1}{3} + 1 = c$

$c = 12 - \frac{1}{4} + \frac{1}{3} = 12 + \frac{1}{12}$

$c = 23/4$

$y^3 + y^2 = \frac{1}{4}x^4 - \frac{1}{3}x^3 + x^2 + \frac{23}{4}$

exact solution