CALCULUS BC
WORKSHEET ON EULER'S METHOD

Work the following on notebook paper, showing all steps.

1. (a) Given the differential equation $\frac{dy}{dx} = x + 2$ and $y(0) = 3$. Find an approximation for $y(1)$ by using Euler's method with two equal steps. Sketch your solution.

   (b) Solve the differential equation $\frac{dy}{dx} = x + 2$ with the initial condition $y(0) = 3$, and use your solution to find $y(1)$.

   (c) The error in using Euler's Method is the difference between the approximate value and the exact value. What was the error in your answer? How could you produce a smaller error using Euler's Method?

2. Suppose a continuous function f and its derivative f' have values that are given in the following table. Given that $f(2) = 5$, use Euler's Method with two steps of size $\Delta x = 0.5$ to approximate the value of $f(3)$.

x	2.0	2.5	3.0
$f'(x)$	0.4	0.6	0.8
$f(x)$	5		

3. Given the differential equation $\frac{dy}{dx} = \frac{1}{x+2}$ and $y(0) = 1$. Find an approximation of $y(1)$ using Euler's Method with two steps and step size $\Delta x = 0.5$.

4. Given the differential equation $\frac{dy}{dx} = x + y$ and $y(1) = 3$. Find an approximation of $y(2)$ using Euler's Method with two equal steps.

5. The curve passing through $(2, 0)$ satisfies the differential equation $\frac{dy}{dx} = 4x + y$. Find an approximation to $y(3)$ using Euler's Method with two equal steps.

6. Assume that f and f' have the values given in the table. Use Euler's Method with two equal steps to approximate the value of $f(4.4)$.

x	4	4.2	4.4
$f'(x)$	-0.5	-0.3	-0.1
$f(x)$	2		

TURN->>

7. The table gives selected values for the derivative of a function f on the interval $-2 \le x \le 2$. If $f(-2) = 3$ and Euler's method with a step-size of 1.5 is used to approximate $f(1)$, what is the resulting approximation?

x	$f'(x)$
-2	-0.8
-1.5	-0.5
-1	-0.2
-0.5	0.4
0	0.9
0.5	1.6
1	2.2
1.5	3
2	3.7

8. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = x + 2y$ with the initial condition $f(0) = 1$. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f(-0.6)$.

Answers

1. (a) $5\frac{1}{4}$

   (b) $5\frac{1}{2}$

   (c) Error $= \frac{1}{4}$. Use smaller steps.

2. 5.5
3. 1.45
4. 8.25
5. 11
6. 1.84
7. 2.4
8. 0.25

Euler's Method

1a $\frac{dy}{dx} = x + 2, \quad y(0) = 3$

x	y	dy/dx	
0	3	2	$\rightarrow \quad y - 3 = 2(x - 0)$
$\frac{1}{2}$	4	2.5	$\rightarrow \quad y - 4 = 2.5(x - \frac{1}{2})$
1	5.25		

$y(1) \approx 5.25$

4b. $\frac{dy}{dx} = x + 2, \quad y(0) = 3$

separate → $dy = (x+2)\,dx$

integrate → $\int dy = \int (x+2)\,dx$

$y = \frac{1}{2}x^2 + 2x + c$

evaluate → $y(0) = 0 + 0 + 3 = 3$

$c = 3$

$y = \frac{1}{2}x^2 + 2x + 3$

$y(1) = \frac{1}{2} + 2 + 3 = 5.5$

---

c. $5.25 - 5.5 = -0.25$ → error

use smaller steps to produce smaller error

2. $f(2) = 5$, $f'(2) = 0.4$

x	y	dy/dx
2	5	0.4
2.5	5.2	0.6
3	5.5	

$\rightarrow y - 5 = 0.4(x - 2)$

$\rightarrow y - 5.2 = 0.6(x - 2.5)$

$f(3) \approx 5.5$

---

3. $\frac{dy}{dx} = \frac{1}{x+2}$, $y(0) = 1$

x	y	dy/dx
0	1	$1/2$
$1/2$	1.25	$2/5$
1	1.45	

$\rightarrow y - 1 = 1/2(x - 0)$

$\rightarrow y - 1.25 = 2/5(x - 1/2)$

$y(1) \approx 1.45$

---

4. $\frac{dy}{dx} = x + y$, $y(1) = 3$

x	y	dy/dx
1	3	4
1.5	5	6.5
2	8.25	

$\rightarrow y - 3 = 4(x - 1)$

$\rightarrow y - 5 = 6.5(x - 1.5)$

$y(2) \approx 8.25$

5. $y(2) = 0$, $\frac{dy}{dx} = 4x + y$

x	y	dy/dx	
2	0	8	→ $y - 0 = 8(x - 2)$
2.5	4	14	→ $y - 4 = 14(x - 2.5)$
3	11		

$y(3) \approx 11$

---

6. $f(4) = 2$.

x	y	dy/dx	
4	2	-0.5	→ $y - 2 = -0.5(x - 4)$
4.2	1.9	-0.3	→ $y - 1.9 = -0.3(x - 4.2)$
4.4	1.84		

$f(4.4) \approx 1.84$

---

7. $f(-2) = 3$

x	y	dy/dx	
-2	3	-0.8	→ $y - 3 = -0.8(x + 2)$
$-1/2$	1.8	0.4	→ $y - 1.8 = 0.4(x + 1/2)$
1	2.4		

$f(1) \approx 2.4$

8. $\frac{dy}{dx} = x + 2y$, $f(0) = 1$

x	y	dy/dx	
0	1	2	$\rightarrow y - 1 = 2(x - 0)$
-0.3	0.4	0.5	$\rightarrow y - 0.4 = 0.5(x + 0.3)$
-0.6	0.25		

$f(-0.6) \approx 0.25$