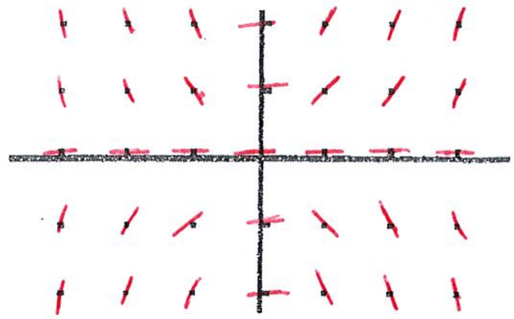
Unit 9 **Test Review**

1. $\dfrac{dy}{dx} = 4xy$

a) Graph the slope field for the given differential equation.



b) Find the *general* solution to the exact differential equation.

$$\int \frac{dy}{y} = \int 4x\,dx$$

$$e^{\ln|y|} = e^{\frac{4x^2}{2} + c}$$

$$y = e^{2x^2 + c} \quad \text{OR} \quad y = Ce^{2x^2}$$

c) Solve the initial value problem explicitly given y(2) = 1

$$1 = e^{2(2)^2 + c}$$

$$\ln 1 = \ln e^{8+c}$$

$$0 = 8 + c$$

$$c = -8$$

$$y = e^{2x^2 - 8}$$

OR

$$1 = Ce^{2(2)^2}$$

$$1 = Ce^{8}$$

$$C = \frac{1}{e^8}$$

$$y = \frac{1}{e^8} \cdot e^{2x^2}$$

d) Write a normal line that goes through the point in part c

$\dfrac{dy}{dx} = 4(2)(1) = 8 \leftarrow$ slope of tangent

so $-\frac{1}{8} \leftarrow$ slope of normal

$$y - 1 = -\frac{1}{8}(x - 2)$$

e) Show that there is a point P with x-coordinate 5 at which the line tangent to the curve at P has a slope of 2.

slope $= 2 \rightarrow \dfrac{dy}{dx} = 4xy$; $x = 5$

$$2 = 4(5)y$$

$$2 = 20y$$

$$y = 1/10$$

point P = $(5, 1/10)$

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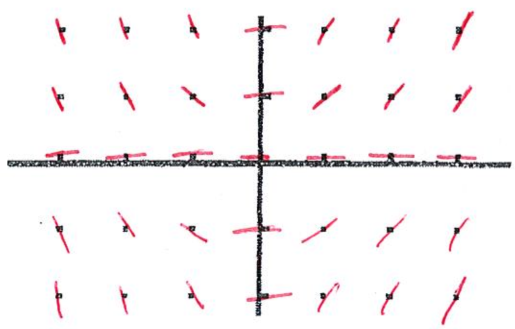
2. $\dfrac{dy}{dx} = 2xy^2$

a) Graph the slope field for the given differential equation.



b) Find the *general* solution to the exact differential equation.

$$\int \frac{dy}{y^2} = \int 2x\,dx$$

$$-\frac{1}{y} = \frac{x^2 + c}{1}$$

$$y = \frac{-1}{x^2 + c}$$

c) Solve the initial value problem explicitly given y(1) = 1

$$1 = \frac{-1}{1^2 + C}$$

$$1 + c = -1$$

$$c = -2$$

$$y = \frac{-1}{x^2 - 2}$$

d) Write a tangent line that goes through the point in part c

slope $= \dfrac{dy}{dx} = 2(1)(1)^2 = 2$

$$y - 1 = 2(x - 1)$$

e) Find $\dfrac{d^2y}{dx^2}$

product rule!

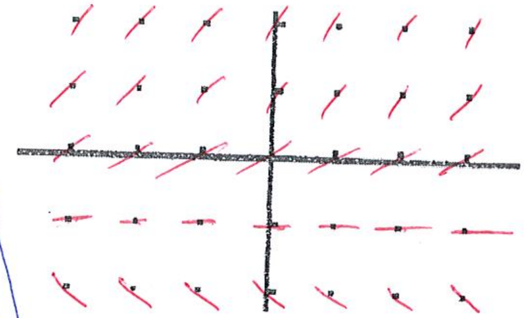
$$\frac{d^2y}{dx^2} = \cancel{4x\left(\frac{dy}{dx}\right) + y^2(2)} = 2x\left(2y\frac{dy}{dx}\right) + y^2 \cdot 2$$

$$\cancel{= 4x(4xy) + y^2(2)}$$

$$2x(2y(2xy^2)) + 2y^2$$

3. $\dfrac{dy}{dx} = y+1$

a) Graph the slope field for the given differential equation. b)



b) Find the *general* solution to the exact differential equation.

$$\int \frac{dy}{y+1} = \int dx$$

$$e^{\ln|y+1|} = e^{x+c}$$

$$y+1 = e^{x+c}$$

$$y = e^{x+c} - 1 \quad \text{or} \quad y = Ce^{x} - 1$$

c) Solve the initial value problem explicitly given y(0) = 2

$$2 = e^{0+c} - 1$$

$$\ln 3 = \ln e^{c}$$

$$c = \ln 3$$

$$y = e^{x+\ln 3} - 1$$

or

$$2 = Ce^{0} - 1$$

$$2 = C - 1$$

$$c = 3$$

$$y = 3e^{x} - 1$$

d) Write a tangent line that goes through the point in part c

$$\text{slope} = \frac{dy}{dx} = 2+1 = 3$$

slope of tangent

$$y - 2 = 3(x - 0)$$

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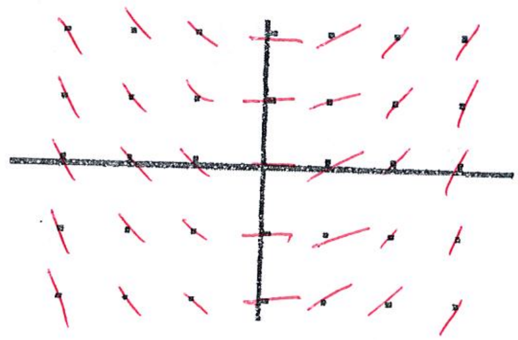
4. $\dfrac{dy}{dx} = 4x$

a) Graph the slope field for the given differential equation.



b) Find the *general* solution to the exact differential equation.

$$\int dy = \int 4x \, dx$$

$$y = \frac{4x^2}{2} + c$$

$$y = 2x^2 + c$$

c) Solve the initial value problem explicitly given y(2) = 1

$$1 = 2(2)^2 + c$$

$$1 = 8 + c$$

$$c = -7$$

$$y = 2x^2 - 7$$

d) Write a normal line that goes through the point in part c

$$\frac{dy}{dx} = 4(2) = 8$$

slope of tangent

so $-\frac{1}{8}$

slope of normal

$$y - 1 = -\frac{1}{8}(x - 2)$$

e) Find $\dfrac{d^2y}{dx^2} = 4$

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5. Find the general solution to the exact differential equation.

a) $\dfrac{dy}{dx} = 6x^5 - \csc^2 x$

$$\int dy = \int 6x^5 - \csc^2 x \, dx$$

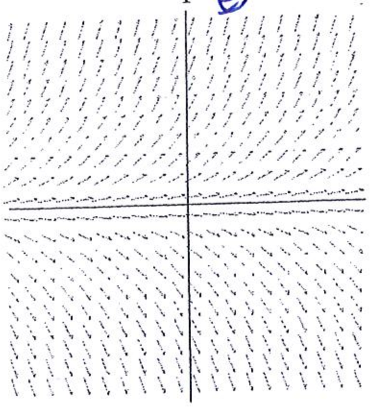
$$y = x^6 + \cot x + c$$

b) $y' = \dfrac{-1}{\sqrt{1-x^2}} - \pi^x \cdot \ln \pi - 5$

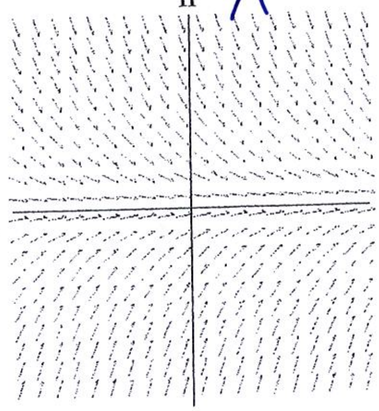
$$y = \arccos x - \pi^x - 5x + c$$

6. Match the slope fields shown below with their differential equations.

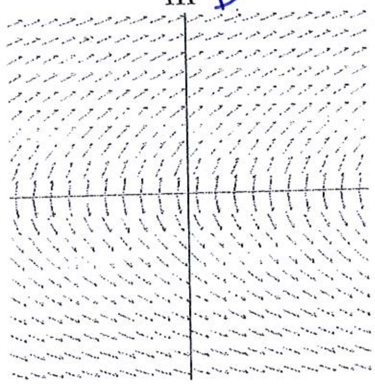
I B



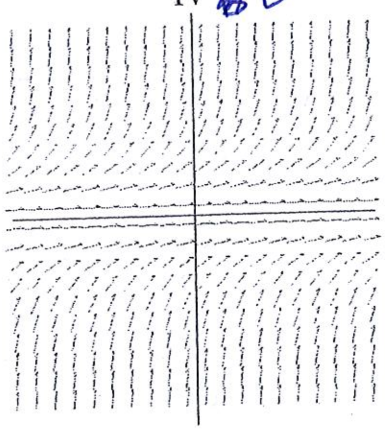
II A



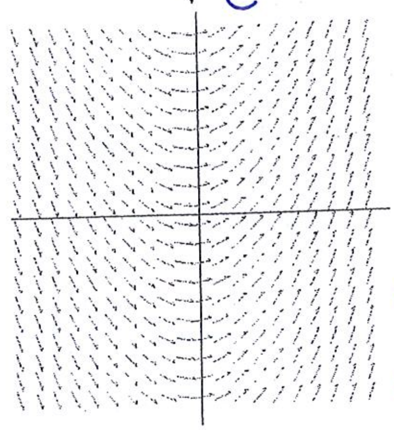
III D



IV ~~B~~ E



V C



you can do this problem 2 ways:

1. solve each diff eq problem to get a general solution. Match general sol. with slope field

2. plug in coordinate pts to diff eq + see which slope field corresponds

best way + easiest way

(A) $\frac{dy}{dx} = -y$ II

(B) $\frac{dy}{dx} = y$ ~~IV~~ I

(C) $\frac{dy}{dx} = x$ V

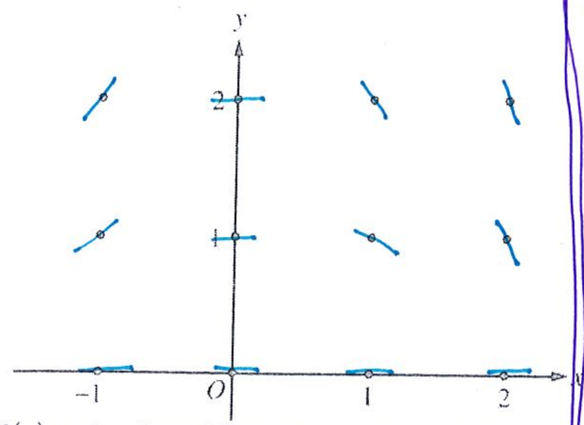
(D) $\frac{dy}{dx} = \frac{1}{y}$ III

(E) $\frac{dy}{dx} = y^2$ IV

7.

Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.
(Note: Use the axes provided in the test booklet.)



$\frac{1}{y^2}\,dy = -\frac{x}{2}\,dx$

$-\frac{1}{y} = -\frac{1}{4}x^2 + c$

$\frac{1}{y} = \frac{1}{4}x^2 + c$

$\boxed{y = \frac{1}{\frac{1}{4}x^2 + c}}$ general solution

(b) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.

(c) Write an equation for the line tangent to the graph of f at $x = -1$.

point $(-1, 2)$

slope: $\frac{dy}{dx} = \frac{-(-1)(2)^2}{2} = 2$

$\boxed{y - 2 = 2(x+1)}$ tangent

$2 = \frac{1}{\frac{1}{4}(-1)^2 + c}$

$2 = \frac{1}{\frac{1}{4} + c}$

$\frac{1}{4} + c = \frac{1}{2}$

$c = 1/4$

solution $\boxed{y = \frac{1}{\frac{1}{4}x^2 + \frac{1}{4}}}$

Multiple Choice

8. If $\cos x = e^y$ and $0 < x < \frac{\pi}{2}$, what is $\frac{dy}{dx}$ in terms of x? (A) $-\tan x$ (B) $-\cot x$ (C) $\cot x$ (D) $\tan x$ (E) $\csc x$

$-\sin x = e^y \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-\sin x}{e^y}$

answer choice not available so manipulate!

note: $\cos x = e^y$

$\frac{dy}{dx} = \frac{-\sin x}{e^y} = \frac{-\sin x}{\cos x} = -\tan x$

9. If $y^2 - 3x = 7$, then $\frac{d^2y}{dx^2} =$ (A) $\frac{-6}{7y^3}$ (B) $\frac{-3}{7y^3}$ (C) 3 (D) $\frac{3}{2y}$ (E) $\frac{-9}{4y^3}$

$2y\frac{dy}{dx} - 3 = 0$

$\frac{dy}{dx} = \frac{3}{2y}$

$\frac{d^2y}{dx^2} = \frac{\cancel{(2y)(0)} - (3)(2\frac{dy}{dx})}{(2y)^2} = -\frac{6(3/2y)}{4y^2} = \frac{-9/y}{4y^2} = \frac{-9}{4y^3}$

sub!

10. If $\tan(x+y) = x$, then $\frac{dy}{dx} =$ (A) $\sec^2(x+y)$ (B) $\ln|\sec(x+y)|$ (C) $\sin^2(x+y) - 1$ (D) $\cos^2(x+y) - 1$

$\sec^2(x+y)(1 + \frac{dy}{dx}) = 1$

$1 + \frac{dy}{dx} = \frac{1}{\sec^2(x+y)}$

$\frac{dy}{dx} = \cos^2(x+y) - 1$

11. If $(x-y)^2 = y^2 - xy$, then $\frac{dy}{dx} =$ (A) $\frac{2x-y}{2y-x}$ (B) $\frac{2x-y}{2x}$ (C) $\frac{2x-y}{x}$ (D) $\frac{2x+3y}{x}$ (E) undefined

$\cancel{2(x-y)\cdot(1 - \frac{dy}{dx}) = 2y\frac{dy}{dx} - (x\frac{dy}{dx} + y)}$

1. foil
2. simplify
3. take derivative

implicit differentiation