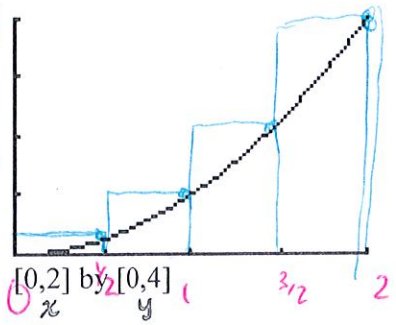
Riemann Sums Worksheet

Estimate each of the following areas using the indicated method with the indicated number of rectangles. Is the approximation an over or under estimation? Be sure to sketch the rectangles in the pictures.

1) $f(x)=x^2$, RRAM, n = 4

$\int_0^2 f(x)\,dx$



$= \frac{1}{2}f(1/2) + \frac{1}{2}f(1) + \frac{1}{2}f(3/2) + \frac{1}{2}f(2)$

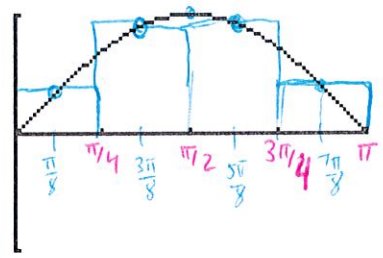
$= \frac{1}{2}\left(f(1/2) + f(1) + f(3/2) + f(2)\right)$

$= \frac{1}{2}\left(1/4 + 1 + 9/4 + 4\right)$

$= \frac{1}{2}\left(30/4\right) = \frac{30}{8} = 3.75$

2) $f(x)=\sin x$, MRAM, n = 4

$\int_0^{\pi} f(x)\,dx$



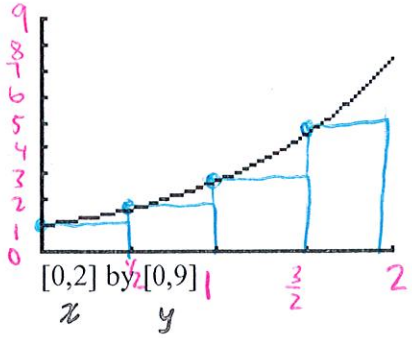
$= \frac{\pi}{4}f(\pi/8) + \frac{\pi}{4}f(3\pi/8) + \frac{\pi}{4}f(5\pi/8) + \frac{\pi}{4}f(7\pi/8)$

$= \frac{\pi}{4}\left(f(\pi/8) + f(3\pi/8) + f(5\pi/8) + f(7\pi/8)\right)$

$= \frac{\pi}{4}\left(\sin\frac{\pi}{8} + \sin\frac{3\pi}{8} + \sin\frac{5\pi}{8} + \sin\frac{7\pi}{8}\right)$

3) $f(x)=e^x$, LRAM, n = 4

$\int_0^2 e^x\,dx$



$= \frac{1}{2}f(0) + \frac{1}{2}f(1/2) + \frac{1}{2}f(1) + \frac{1}{2}f(3/2)$

$= \frac{1}{2}\left(f(0) + f(1/2) + f(1) + f(3/2)\right)$

$= \frac{1}{2}\left(e^0 + e^{1/2} + e^1 + e^{3/2}\right)$

$= \frac{1}{2}\left(1 + \sqrt{e} + e + \sqrt{e^3}\right)$