

CALCULUS  
WORKSHEET ON RIEMANN SUMS AND TRAPEZOIDAL RULE

1. A table of values for  $f(t)$  is given.

$t$	0	20	40	60	80	100	120
$f(t)$	1.2	2.8	4.0	4.7	5.1	5.2	4.8

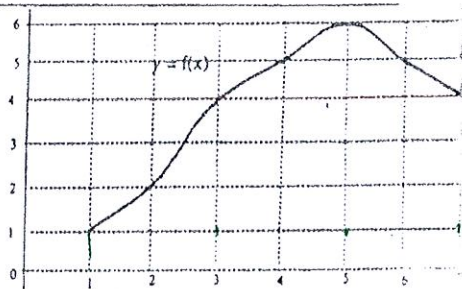
- (a) Estimate  $\int_0^{120} f(t)$  by using a left Riemann sum with six subintervals.  
 (b) Estimate  $\int_0^{120} f(t)$  by using a right Riemann sum with six subintervals.  
 (c) Estimate  $\int_0^{120} f(t)$  by using a midpoint sum with three subintervals.  
 (d) Estimate  $\int_0^{120} f(t)$  by using the trapezoidal rule with three subintervals.

2. A table of values for  $g(t)$  is given.

$t$	0	40	70	90	100
$g(t)$	150	180	195	184	172

- (a) Estimate  $\int_0^{100} g(t) dt$  by using a left Riemann sum with four subintervals.  
 (b) Estimate  $\int_0^{100} g(t) dt$  by using a right Riemann sum with four subintervals.  
 (c) Estimate  $\int_0^{100} g(t) dt$  by using the trapezoidal rule with four subintervals.

3. The graph of the function  $f$  over the interval  $[1, 7]$  is shown. Using values from the graph, find trapezoidal rule estimates for the integral  $\int_1^7 f(x) dx$  by using the indicated number of subintervals.



- (a)  $n = 3$   
 (b)  $n = 6$

① a.  $20(1.2) + 20(2.8) + 20(4) + 20(4.7) + 20(5.1) + 20(5.2) = 460$   
 b.  $20(2.8) + 20(\frac{4.0}{4}) + 20(\frac{4.7}{4.7}) + 20(\frac{5.1}{5.1}) + 20(\frac{5.2}{5.2}) + 20(4.8) = 532$   
 c.  $40(2.8) + 40(4.7) + 40(5.2) = 508$   
 d.  $\frac{1}{2}(40)(1.2 + 4) + \frac{1}{2}(40)(4 + 5.1) + \frac{1}{2}(40)(5.1 + 4.8) = 484$

② a.  $40(150) + 30(180) + 20(195) + 10(184) = 17,140$   
 b.  $40(180) + 40(195) + 40(184) + 40(172) = 29,240$   
 c.  $\frac{1}{2}(40)(150 + 180) + \frac{1}{2}(30)(180 + 195) + \frac{1}{2}(20)(195 + 184) + \frac{1}{2}(10)(184 + 172) = 17,795$

d. a.  $\frac{1}{2}(2)(4+1) + \frac{1}{2}(2)(6+4) + \frac{1}{2}(2)(4+6) = 25$

b.  $\frac{1}{2}(1)(1+2) + \frac{1}{2}(1)(2+4) + \frac{1}{2}(1)(4+5) + \frac{1}{2}(1)(5+6) + \frac{1}{2}(1)(6+5) + \frac{1}{2}(1)(5+4) = 24.5$