HW - p.468 # 1-10 all

1. $\boxed{\int \underset{dv}{x^2} \overset{u}{\ln x}\, dx}$

$u = \ln x$ $\qquad$ $v = \frac{1}{3}x^3$

$du = \frac{1}{x}\,dx$ $\qquad$ $dv = x^2\,dx$

$$\int u\,dv = uv - \int v\,du$$

$$= (\ln x)\left(\tfrac{1}{3}x^3\right) - \int \tfrac{1}{3}x^3 \cdot \tfrac{1}{x}\,dx$$

$$= \tfrac{1}{3}x^3 \ln x - \tfrac{1}{3}\int x^2\,dx$$

$$= \tfrac{1}{3}x^3 \ln x - \tfrac{1}{3}\cdot\tfrac{x^3}{3} + c$$

$$= \boxed{\tfrac{1}{3}x^3 \ln x - \tfrac{x^3}{9} + c}$$

2. $\boxed{\int \overset{u}{\theta}\, \underset{dv}{\cos\theta\, d\theta}}$

$u = \theta$ $\qquad$ $v = \sin\theta$

$du = 1\,d\theta$ $\qquad$ $dv = \cos\theta\,d\theta$

$$\int u\,dv = uv - \int v\,du$$

$$= \theta\sin\theta - \int \sin\theta \cdot d\theta$$

$$= \boxed{\theta\sin\theta + \cos\theta + c}$$

3. $\boxed{\int \overset{u}{x \cos 5x}\, \underset{dv}{dx}}$

$u = \cos 5x$ $\qquad$ $v = \frac{1}{2}x^2$

$du = -5\sin 5x\,dx$ $\qquad$ $dv = \cancel{x\,dx}$

$$\int u\,dv = uv - \int v\,du$$

$$= \cos 5x \cdot \tfrac{1}{2}x^2 - \int \tfrac{1}{2}x^2 \cdot -5\sin 5x\,dx$$

OOPS!

3. $\boxed{\int x \cos 5x \, dx}$

$u = x \qquad v = \frac{1}{5}\sin 5x$

$du = dx \qquad dv = \cos 5x \, dx$

$$\int u\,dv = uv - \int v\,du$$

$$= x \cdot \frac{1}{5}\sin 5x - \int \frac{1}{5}\sin 5x \cdot dx$$

$$= \boxed{x \cdot \frac{1}{5}\sin 5x + \frac{1}{25}\cos 5x + c}$$

4. $\boxed{\int y e^{0.2y} \, dy}$

$u = y \qquad v = 5e^{1/5 y}$

$du = dy \qquad dv = e^{1/5 y}\, dy$

$$\int u\,dv = uv - \int v\,du$$

$$= y \cdot 5e^{1/5 y} - \int 5e^{1/5 y}\, dy$$

$$= \boxed{5ye^{1/5 y} - 25e^{1/5 y} + c}$$

5. $\boxed{\int t e^{-3t} \, dt}$

$u = t \qquad v = -\frac{1}{3}e^{-3t}$

$du = dt \qquad dv = e^{-3t}\, dt$

$$\int u\,dv = uv - \int v\,du$$

$$= t \cdot -\frac{1}{3}e^{-3t} - \int -\frac{1}{3}e^{-3t}\, dt$$

$$= \boxed{-\frac{t}{3}e^{-3t} - \frac{1}{9}e^{-3t} + c}$$

6. $\boxed{\int (x-1) \sin \pi x \, dx}$

$u = x-1 \qquad v = -\frac{1}{\pi}\cos \pi x$

$du = dx \qquad dv = \sin \pi x \, dx$

$$\int u\,dv = uv - \int v\,du$$

$$= (x-1)\left(-\frac{1}{\pi}\cos \pi x\right) - \int -\frac{1}{\pi}\cos \pi x \, dx$$

$$= (x-1)\left(-\frac{1}{\pi}\cos \pi x\right) + \frac{1}{\pi}\int \cos \pi x \, dx$$

$$= \boxed{(x-1)\left(-\frac{1}{\pi}\cos \pi x\right) + \frac{1}{\pi^2}\sin \pi x + C}$$

7. $\boxed{\int (x^2+2x)\cos x \, dx}$

$u = x^2+2x \qquad v = \sin x$

$du = (2x+2)\,dx \qquad dv = \cos x \, dx$

$$\int u\,dv = uv - \int v\,du$$

$$= (x^2+2x)\sin x - \int \sin x\,(2x+2)\,dx$$

again!

$u = 2x+2$

$du = 2\,dx$

$v = -\cos x$

$dv = \sin x$

$$= (x^2+2x)\sin x - \left[(2x+2)(-\cos x) - \int -2\cos x \, dx\right]$$

$$= \boxed{(x^2+2)\sin x + (2x+2)\cos x - 2\sin x + C}$$

$$= x^2\sin x + (2x+2)\cos x + C$$

8. $\boxed{\int t^2 \sin \beta t \, dt}$

$u = t^2 \qquad v = -\frac{1}{\beta}\cos \beta t$

$du = 2t\,dt \qquad dv = \sin \beta t \, dt$

$$\int u\,dv = uv - \int v\,du$$

$$= t^2 \cdot -\frac{1}{\beta}\cos \beta t - \int -\frac{1}{\beta}\cos \beta t \cdot 2t\,dt$$

$$= -\frac{t^2}{\beta}\cos \beta t + \frac{2}{\beta}\int \cos \beta t \cdot \cancel{2}t\,dt$$

again!

$u = \cancel{2}t$

$du = \cancel{2}\,dt$

$v = \frac{1}{\beta}\sin \beta t$

$dv = \cos \beta t$

$$= -\frac{t^2}{\beta}\cos \beta t + \frac{2}{\beta}\left(t \cdot \frac{1}{\beta}\sin \beta t - \int \frac{1}{\beta}\sin \beta t \, dt\right)$$

$$= \boxed{-\frac{t^2}{\beta}\cos \beta t + \frac{2}{\beta}\left(t \cdot \frac{1}{\beta}\sin \beta t + \frac{1}{\beta^2}\cos \beta t + C\right)}$$

9. $\boxed{\int \ln\sqrt[3]{x}\,dx}$ ($u = \ln\sqrt[3]{x}$, $dv = dx$)

$u = \ln\sqrt[3]{x}$ $\quad\quad$ $v = x$

$du = \frac{1}{x^{1/3}} \cdot \frac{1}{3} x^{-2/3}\,dx$ $\quad\quad$ $dv = dx$

$$u\,dv = uv - \int v\,du$$

$$= x\ln\sqrt[3]{x} - \int x \cdot \frac{1}{x^{1/3}} \cdot \frac{1}{3} \cdot \frac{x^{-2/3}}{1}$$

$$= x\ln\sqrt[3]{x} - \int \frac{1}{3}\,dx$$

$$= \boxed{x\ln\sqrt[3]{x} - \frac{1}{3} \cdot x + c}$$

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10. $\boxed{\int \sin^{-1}x\,dx}$ ($u = \sin^{-1}x$, $dv = dx$)

$u = \sin^{-1}x$ $\quad\quad$ $v = x$

$du = \frac{1}{\sqrt{1-x^2}}\,dx$ $\quad\quad$ $dv = dx$

$$u\,dv = uv - \int v\,du$$

$$= x\sin^{-1}x - \int \underbrace{x \cdot \frac{1}{\sqrt{\underbrace{1-x^2}_{u}}}\,dx}_{-1/2\,du}$$

$$= x\sin^{-1}x - \int u^{-1/2}\,du$$

$$= x\sin^{-1}x - 2u^{1/2} + c$$

$$= x\sin^{-1}x - 2(1-x^2)^{1/2} + c$$

$$= \boxed{x\sin^{-1}x - 2\sqrt{1-x^2} + c}$$

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$