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7. $\int \frac{x^4}{x-1}\,dx$

$= \int x^3 + x^2 + x + 1 + \frac{1}{x-1}\,dx$

$$\boxed{= \frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x-1| + c}$$

$$x-1 \,\overline{)\,x^4}\quad x^3 + x^2 + x + 1 + \frac{1}{x-1}$$

$$\underline{-x^4 + x^3}$$

$$x^3$$

$$\underline{-x^3 + x^2}$$

$$x^2$$

$$\underline{-x^2 + x}$$

$$x$$

$$\underline{-\ x + 1}$$

$$1$$

---

8. $\int \frac{3t-2}{t+1}\,dt$

$= \int 3 - \frac{5}{t+1}\,dt$

$$\boxed{= 3t - 5\ln|t+1| + c}$$

$$t+1 \,\overline{)\,3t-2}\quad 3 - 5/t+1$$

$$\underline{-3t \overset{-}{+} 3}$$

$$-5$$

---

9. $\int \frac{5x+1}{(2x+1)(x-1)}\,dx$

$= \int \frac{1}{2x+1} + \frac{2}{x-1}\,dx$

$$\boxed{= \frac{1}{2}\ln|2x+1| + 2\ln|x-1| + c}$$

$$\frac{5x+1}{(2x+1)(x-1)} = \frac{A}{(2x+1)} + \frac{B}{(x-1)}$$

$$5x+1 = A(x-1) + B(2x+1)$$

$$= Ax - A + 2Bx + B$$

$$5x+1 = (A+2B)x + B - A$$

$$5 = A + 2B$$

$$1 = -A + B$$

$$6 = 3B$$

$$B = 2$$

$$A = 1$$

10. $\int \frac{y}{(y+4)(2y-1)}\,dy$

$$= \int \frac{4/9}{y+4} + \frac{\frac{1}{9}}{2y-1}\,dy$$

$$\boxed{= \frac{4}{9}\ln|y+4| + \frac{1}{18}\ln|2y-1| + c}$$

$$\frac{y}{(y+4)(2y-1)} = \frac{A}{y+4} + \frac{B}{2y-1}$$

$y = A(2y-1) + B(y+4)$

$y = 2yA - A + By + 4B$

$y = (B+2A)y - A + 4B$

$$\left.\begin{array}{l} B+2A=1 \\ 4B-A=0 \end{array}\right> \begin{array}{l} B+2A=1 \\ 8B-2A=0 \end{array}$$

$9B = 1$

$B = 1/9$

$A = 4/9$

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31. $\int_0^{1/2} \cos^{-1} x \, dx$

$u = \arccos x$
$du = \frac{-1}{\sqrt{1-x^2}} dx$

$v = x$
$dv = dx$

$= x \arccos x - \int_0^{1/2} x \cdot \frac{-1}{\sqrt{1-x^2}} dx$

$u = 1-x^2$
$du = -2x \, dx$
$-\frac{1}{2} dx = x \, dx$

$= x \arccos x + \frac{-1}{2} \int \frac{1}{u^{1/2}} du$

$= x \arccos x - \frac{1}{\cancel{2}} \cdot \cancel{2} u^{1/2} + c$

$= x \arccos x - (1-x^2)^{1/2} + c \Big]_0^{1/2}$

$= \left[\frac{1}{2} \arccos \frac{1}{2} - \sqrt{(1-(1/2)^2)}\right] - \left[0 - \sqrt{1-(0)^2}\right]$

$= \frac{1}{2} \cdot \frac{\pi}{3} - \sqrt{3/4} + 1 = \pi/6 - \frac{\sqrt{3}}{\sqrt{4}} + 1 = \boxed{\pi/6 - \sqrt{3}/2 + 1}$

---

32. $\int_1^2 \frac{(\ln x)^2}{x^3} dx = \int_1 x^{-3} (\ln x)^2 dx$

$u = (\ln x)^2$
$du = 2(\ln x) \cdot \frac{1}{x} dx$

$v = -\frac{1}{2} x^{-2}$
$dv = x^{-3} dx$

$= -\frac{1}{2} x^{-2} \cdot (\ln x)^2 - \int -\frac{1}{2} x^{-2} \cdot 2 \ln x \cdot \frac{1}{x} dx$

$= \frac{-(\ln x)^2}{2x^2} + \int \frac{\ln x}{x^3} dx$

$u = \ln x$
$du = \frac{1}{x} dx$

$v = -\frac{1}{2} x^{-2}$
$dv = x^{-3} dx$

$= \frac{-(\ln x)^2}{2x^2} + \ln x \cdot -\frac{1}{2} x^{-2} - \int -\frac{1}{2} x^{-2} \cdot \frac{1}{x} dx$

$= \frac{-(\ln x)^2}{2x^2} - \frac{\ln x}{2x^2} + \frac{1}{2} \int x^{-3} dx = \frac{-(\ln x)^2}{2x^2} - \frac{\ln x}{2x^2} + \frac{1}{2} \cdot \frac{x^{-2}}{-2} \Big]_1^2$

$= \left[\frac{-(\ln 2)^2}{2 \cdot 2^2} - \frac{\ln 2}{2 \cdot 2^2} \cancel{+} - \frac{1}{4 \cancel{x} \cdot 2^2}\right] - \left[0 - 0 \cancel{+} \frac{1}{4 \cdot 1^2}\right]$

$= -(\ln 2)^2 - \ln 2 - 1 + 1 = \boxed{-(\ln 2)^2 - \ln 2 + 3/}$