1. $\displaystyle\int_0^1 \frac{2}{2x^2+3x+1}\,dx$

$$\frac{2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$$

$$2 = A(x+1) + B(2x+1)$$
$$2 = Ax + A + 2Bx + B$$
$$2 = \underbrace{(A+2B)}_{0}x + \underbrace{A+B}_{2}$$

$A + 2B = 0$
$A + B = 2 \rightarrow \boxed{A = 2 - B}$

$(2-B) + 2B = 0$
$2 + B = 0$
$B = -2$ and $A = 4$

$$\int_0^1 \frac{4}{2x+1} + \frac{-2}{x+1}\,dx$$
$$= 4\cdot\tfrac{1}{2}\ln|2x+1| - 2\ln|x+1|\Big]_0^1$$
$$= [2\ln 3 - 2\ln 2] - \cancel{[2\ln 1 - 2\ln 1]}$$
$$= \boxed{2\ln(3/2)}$$

---

2. $\displaystyle\int_1^2 x^5 \ln x\,dx$

$u = \ln x$ $\qquad v = \frac{1}{6}x^6$
$du = \frac{1}{x}\,dx$ $\qquad dv = x^5\,dx$

$$= (\ln x)(\tfrac{1}{6}x^6) - \int_1^2 (\tfrac{1}{6}x^6)(\tfrac{1}{x})\,dx$$
$$= \tfrac{1}{6}x^6\ln x - \tfrac{1}{6}\int_1^2 x^5\,dx$$
$$= \tfrac{1}{6}x^6\ln x - \tfrac{1}{6}\cdot\tfrac{1}{6}x^6\Big]_1^2 = \tfrac{1}{6}x^6\ln x - \tfrac{1}{36}x^6\Big]_1^2$$
$$= [\tfrac{1}{6}\cdot 64\cdot\ln 2 - \tfrac{1}{36}\cdot 64] - [\tfrac{1}{6}\underbrace{\cancel{\ln 1}}_{0} - \tfrac{1}{36}\cdot 1]$$
$$= [\tfrac{32}{3}\ln(2) - \tfrac{16}{9} + \tfrac{1}{36}] = \boxed{\tfrac{32}{3}\ln(2) - 7/4}$$

---

3. $\displaystyle\int_0^1 \frac{x-4}{x^2-5x+6}\,dx$

$$\frac{x-4}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

$$x-4 = A(x-2) + B(x-3)$$
$$x-4 = Ax - 2A + Bx - 3B$$
$$x-4 = \underbrace{(A+B)}_{1}x - \underbrace{2A-3B}_{-4}$$

$A + B = 1$
$-2A - 3B = -4$ $\rightarrow$ $A = -1$, $B = 2$

$$\int_0^1 \frac{-1}{x-3} + \frac{2}{x-2}\,dx$$
$$= -\ln|x-3| + 2\ln|x-2|\Big]_0^1$$
$$= [-\ln|-2| + 2\underbrace{\cancel{\ln|-1|}}_{0}] - [-\ln|-3| + 2\ln|-2|]$$
$$= -\ln 2 + \ln 3 + 2\ln 2$$
$$= -\ln 2 + \ln 3 + \ln 4$$
$$= \boxed{\ln 3/8}$$

$$\int \frac{ax}{x^2-bx}\,dx$$

$$\int \frac{a\cancel{x}}{\cancel{x}(x-b)}\,dx = \int \frac{a}{x-b}\,dx = a\int \frac{1}{x-b}\,dx$$

$$= \boxed{a\ln|x-b|+c}$$

---

5. $$\int_0^2 \frac{dt}{(1-6t)^4}$$

$u = 1-6t$

$du = -6\,dt$

$-\frac{1}{6}du = dt$

$-\frac{1}{6}du$, u

* error - function do not continuous / differentiable on closed interval ∴

$$= -\frac{1}{6}\int_1^{-11} \frac{1}{u^4}\,du = \frac{1}{6}\int_{-11}^{1} u^{-4}\,du$$

$$= \frac{1}{6}\cdot -\frac{1}{3}u^{-3}\Big]_{-11}^{1} = -\frac{1}{18u^3}\Big]_{-11}^{1}$$

$$= \left[\frac{-1}{18}\right] - \left[\frac{-1}{18(-11)^3}\right] = \boxed{\frac{-74}{1331}}$$

---

6. $$\int_5^{10} \frac{x^2+2}{x+2}\,dx$$

$$\boxed{\frac{x^2+2}{x+2}} = x+2\,\overline{)\,x^2+2\,}\ \ (x-2) = \boxed{x-2+\frac{6}{x+2}}$$

$$\begin{array}{r} x-2 \\ x+2\,\overline{)\,x^2+2} \\ -x^2 \mp 2x \\ \hline -2x+2 \\ +2x+4 \\ \hline 6 \end{array}$$

$$\int_5^{10} x-2+\frac{6}{x+2}\,dx$$

$$= \frac{1}{2}x^2 - 2x + 6\ln|x+2|\Big]_5^{10}$$

$$= \left[\frac{1}{2}(10)^2 - 2(10) + 6\ln|12|\right] - \left[\frac{1}{2}(5)^2 - 2(5) + 6\ln|7|\right]$$

$$= [30 + 6\ln 12] - [5/2 + 6\ln 7] = \boxed{\frac{55}{2} + 6\ln(12/7)}$$