p. 527 #1-2, 5-11

explain why improper

1. a. $\int_1^2 \frac{x}{x-1}\,dx$ not cont. at $x=1$

b. $\int_0^\infty \frac{1}{1+x^2}\,dx$ has infinite bound

c. $\int_{-\infty}^{\infty} x^2 e^{-x^2}\,dx$ has infinite bounds

d. $\int_0^{\pi/4} \cot x\,dx$ not cont. at $x=0$

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which are improper? why?

2. a. $\int_0^{\pi/4} \tan x\,dx$ proper → $\tan x$ defined everywhere on $[0,\pi/4]$

b. $\int_0^{\pi} \tan x\,dx$ improper → $\tan x$ not defined at $x=\pi/2$

c. $\int_{-1}^{1} \frac{dx}{\underbrace{x^2-x-2}_{(x-2)(x+1)}}$ improper → not defined at $x=-1$

d. $\int_0^\infty e^{-x^3}\,dx$ improper → infinite bound

5. $\displaystyle\int_3^\infty \frac{1}{(x-2)^{3/2}}\,dx = \lim_{t\to\infty}\int_3^t \frac{1}{((x-2))^{3/2}}\frac{du}{dx}$ $\boxed{\begin{array}{l} u = x-2 \\ du = dx \end{array}}$

$\displaystyle = \lim_{t\to\infty}\int u^{-3/2}\,du = \lim_{t\to\infty} -2(u)^{-1/2}\,du = \lim_{t\to\infty} -2(x-2)^{-1/2}\Big]_3^t$

$\displaystyle = \lim_{t\to\infty}\left[-2(t-2)^{-1/2} + 2(3-2)^{-1/2}\right] = \lim_{t\to\infty}\frac{-2}{\sqrt{t-2}} + 2\cdot 1$

$\displaystyle = \frac{-2}{\sqrt{\infty-2}} + 2 = \frac{-2}{\infty}^{\,0} + 2 = \boxed{2}$ convergent

6. $\displaystyle\int_0^\infty \frac{1}{(1+x)^{1/4}}\,dx = \lim_{t\to\infty}\int_0^t (1+x)^{-1/4}\,dx = \lim_{t\to\infty}\frac{4}{3}(1+x)^{3/4}\Big]_0^t$

$\displaystyle = \lim_{t\to\infty}\left(\frac{4}{3}(1+t)^{3/4} - \frac{4}{3}(1+0)^{3/4}\right] = \lim_{t\to\infty}\frac{4}{3}(1+t)^{3/4} - \frac{4}{3}$

$\displaystyle \lim_{t\to\infty}\frac{4}{3}(1+\infty)^{3/4} = \infty$ DIVERGENT

7. $\displaystyle\int_{-\infty}^0 \frac{1}{3-4x}\,dx = \lim_{t\to-\infty}\int_t^0 \frac{1}{3-4x}\,dx = -\frac{1}{4}\ln|3-4x|\Big]_t^0$

$\displaystyle \lim_{t\to-\infty}\left(-\frac{1}{4}\ln 3\right) - \left(-\frac{1}{4}\ln|3-4t|\right) = -\frac{1}{4}\ln 3 + \frac{1}{4}\ln|3-4\infty|$

divergent

8. $\int_1^\infty \frac{1}{(2x+1)^3}\,dx = \lim_{t\to\infty} \int_1^t \frac{1}{(2x+1)^3}$

$u = 2x+1$
$du = 2\,dx$
$\frac{1}{2}du = dx$

$= \lim_{t\to\infty} \frac{1}{2}\cdot\frac{1}{-2}(2x+1)^{-2}\Big]_1^t = -\frac{1}{4(2x+1)^2}\Big]_1^t$

$= \lim_{t\to\infty} \left(-\frac{1}{4(2t+1)^2}\right) - \left(-\frac{1}{4(2\cdot1+1)^2}\right) = \frac{-1}{4(2t+1)^2} + \frac{1}{36}$

$\lim_{t\to\infty} \frac{1}{36} - \frac{1}{4(2\infty+1)^2} = \frac{1}{36} + 0$ convergent

9. $\int_2^\infty e^{-5p}\,dp = \lim_{t\to\infty} \int_2^t e^{-5p}\,dp = \lim_{t\to\infty} -\frac{1}{5}e^{-5p}\Big]_2^t$

$= \left(-\frac{1}{5}e^{-5t}\right) - \left(-\frac{1}{5}e^{-5\cdot2}\right) = -\frac{1}{5}e^{-5\cdot\infty} + \frac{1}{5}e^{-10}$

$= \frac{-1}{5e^\infty} + \frac{1}{5e^{10}} = 0 + \boxed{\frac{1}{5e^{10}}}$ convergent

10. $\int_{-\infty}^0 2^r\,dr = \lim_{t\to-\infty} \int_t^0 2^r\,dr = \frac{1}{\ln 2}\cdot 2^r\Big]_t^0$

$= \lim_{t\to\infty} \left(\frac{1}{\ln 2}\cdot 2^0\right) - \left(\frac{1}{\ln 2}\cdot 2^t\right) = \frac{1}{\ln 2} - \frac{1}{\ln 2}\cdot 2^{-\infty}$

$= \frac{1}{\ln 2} - \frac{1}{2^\infty\cdot\ln 2} = \frac{1}{\ln 2}$ convergent

11. $\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}\,dx = \lim_{t\to\infty} \int_0^t \frac{x^2}{\sqrt{1+x^3}}\,dt$

$u = 1+x^3$
$du = 3x^2\,dx$
$\frac{1}{3}du = x^2\,dx$

$\frac{1}{3}\int u^{-1/2}\,du$
$= \frac{1}{3}\cdot 2u^{1/2}$

$= \lim_{t\to\infty} \frac{2}{3}(1+x^3)^{1/2}\Big]_0^t$

$= \frac{2}{3}(1+t^3)^{1/2} - \frac{2}{3}(1+0^3)^{1/2} = \lim_{t\to\infty} \frac{2}{3}(1+\infty^3)^{1/2} - \frac{2}{3}$ divergent