Test Review Find the antiderivative.

1. $\int \frac{2x-5}{(x^2-5x)^3}dx$

$u = x^2 - 5x$

$du = 2x - 5\ dx$

$= \int u^{-3}\,du$

$= -\frac{1}{2}u^{-2} + c$

$= \boxed{-\frac{1}{2}(x^2-5x)^{-2} + c}$

2. $\int x\ln x\,dx$

$u = \ln x \qquad v = \frac{1}{2}x^2$

$du = \frac{1}{x}dx \qquad dv = x\,dx$

$= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$

$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx$

$= \boxed{\frac{1}{2}x^2\ln x - \frac{1}{4}x^2 + c}$

3. $\int \frac{4t}{t^2+t-2}dt$

$\frac{4t}{(t+2)(t-1)} = \frac{A}{t+2} + \frac{B}{t-1}$

$4t = A(t-1) + B(t+2)$

$4t = \underbrace{(A+B)}_{4}t\ \underbrace{-A+2B}_{0}$

$A+B=4$

$-A+2B=0$

$B = 4/3$

$A = 8/3$

$\int \frac{8/3}{t+2} + \frac{4/3}{t-1}\cdot dt$

$= \boxed{\frac{8}{3}\ln|t+2| + \frac{4}{3}\ln|t-1| + c}$

4. $\int \frac{x^3-3x^2+1}{x^2+1}dx$

$$
\begin{array}{r}
x - 3 + \frac{-x+4}{x^2+1} \\
x^2+1\,\overline{)\,x^3-3x^2+0x+1} \\
-x^3 \qquad\quad \mp x \\
\hline
-3x^2 - x \\
\pm 3x^2 \qquad \pm 3 \\
\hline
-x+3+1
\end{array}
$$

$= \int x - 3 + \frac{-x+4}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}x^2 - 3x + \int \frac{-x}{x^2+1}dx + \int \frac{4}{x^2+1}dx$

$= \frac{1}{2}x^2 - 3x + 4\arctan x - \frac{1}{2}\int \frac{1}{u}du$

$= \frac{1}{2}x^2 - 3x + 4\arctan x - \frac{1}{2}\ln|u| + c$

$= \boxed{\frac{1}{2}x^2 - 3x + 4\arctan x - \frac{1}{2}\ln|x^2+1| + c}$

5. $\int -\sec^2(3x)(dx)$

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= -\frac{1}{3}\int \sec^2 u\,du$

$= -\frac{1}{3}\tan u + c$

$= \boxed{-\frac{1}{3}\tan(3x) + c}$

6. $\int \frac{4}{3x+1}dx$

$u = 3x+1$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{4}{3}\int \frac{1}{u}du$

$= \frac{4}{3}\ln|u| + c$

$= \boxed{\frac{4}{3}\ln|3x+1| + c}$

7. $\int x^2 \sin x\,dx$

$u = x^2 \qquad v = -\cos x$

$du = 2x\,dx \qquad dv = \sin x\,dx$

$= -x^2\cos x - \int -\cos x \cdot 2x\,dx$

$= -x^2\cos x + \int \cos x \cdot 2x\,dx$

$u = 2x \qquad v = \sin x$

$du = 2\,dx \qquad dv = \cos x\,dx$

$= -x^2\cos x + 2x\cdot\sin x - \int \sin x \cdot 2\,dx$

$= -x^2\cos x - 2x\cdot\sin x + 2\cos x$

$= \boxed{-x^2\cos x + 2x\cdot\sin x + 2\cos x + c}$

8. $\int \frac{a+bx^2}{\sqrt{3ax+bx^3}}dx$

$u = 3ax + bx^3$

$du = 3a + 3bx^2\,dx$

$\frac{1}{3}du = a + bx^2\,dx$

$= \frac{1}{3}\int \frac{1}{\sqrt{u}}du$

$= \frac{1}{3}\int u^{-1/2}du$

$= \frac{2}{3}u^{1/2} + c$

$= \boxed{\frac{2}{3}(3ax+bx^3)^{1/2} + c}$

9. $\int_2^\infty xe^{-2x}\,dx = \lim_{t\to\infty} \int_2^t xe^{-2x}\,dx$

$u = x \qquad v = -\tfrac{1}{2}e^{-2x}$

$du = dx \qquad dv = e^{-2x}\,dx$

$= \lim_{t\to\infty} x\cdot -\tfrac{1}{2}e^{-2x} - \int -\tfrac{1}{2}e^{-2x}\,dx$

$-\frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$

$-\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x}\Big]_2^t$

L'Hospital's

$\left[\frac{-t}{2e^{2t}} - \frac{1}{4}e^{-2t}\right] - \left[-e^{-4} - \frac{1}{4}e^{-4}\right]$

$\frac{-1}{4e^{2t}} - \frac{1}{4}e^{-2t} + \frac{5}{4}e^{-4}$

$= \cancel{\frac{-1}{4e^{2\infty}}}^{0} - \cancel{\frac{1}{4e^{2\infty}}}^{0} + 5/4\,e^{-4} = \boxed{5/4\,e^{-4}}$ converges

10. $\int_{-\infty}^\infty xe^{-x^2}\,dx = \lim_{t\to-\infty}\int_t^0 xe^{-x^2}\,dx + \lim_{t\to\infty}\int_0^t xe^{-x^2}\,dx$

$u = -x^2$

$du = -2x\,dx$

$-\tfrac{1}{2}du = x\,dx$

$-\frac{1}{2}\int_{-t^2}^{0} e^u\,du + -\frac{1}{2}\int_0^{-t^2} e^u\,du$

$-\frac{1}{2}e^u\Big]_{-t^2}^{0} + -\frac{1}{2}e^u\Big]_0^{-t^2}$

$-\frac{1}{2}\left[e^0 - e^{-t^2}\right] + -\frac{1}{2}\left[e^{-t^2} - e^0\right]$

$-\cancel{\frac{1}{2}} + \frac{e^{-(-\infty)^2}}{2} + -\frac{1}{2}e^{-\infty^2} + \cancel{\frac{1}{2}}$

$\cancel{\frac{1}{2e^\infty}}^{0} + \cancel{\frac{-1}{2e^\infty}}^{0} = \boxed{0}$ converges

Find the exact length of the curve.

11. $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad 1 \le x \le 2$

$\frac{dy}{dx} = \frac{1}{2}x - \frac{1}{2x}$

$\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{4} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4x^2}$

$= \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}\right)}\,dx = \int_1^2 \sqrt{\frac{1}{2} + \frac{x^2}{4} + \frac{1}{4x^2}}\,dx$

$= \int_1^2 \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2}\,dx = \int_1^2 \frac{x}{2} + \frac{1}{2x}\cdot dx$

$= \frac{1}{4}x^2 + \frac{1}{2}\ln|x|\Big]_1^2$

$= \left[\frac{1}{4}(2)^2 + \frac{1}{2}\ln 2\right] - \left[\frac{1}{4}(1)^2 + \frac{1}{2}\cancel{\ln|1|}^{0}\right]$

$= 1 + \frac{1}{2}\ln 2 - \frac{1}{4} = \boxed{3/4 + 1/2\ln 2}$

12. $y^2 = 4(x+4)^3, \quad 0 \le x \le 2, \quad y > 0$

$y = 2(x+4)^{3/2}$

$\frac{dy}{dx} = 3(x+4)^{1/2}$

$\left(\frac{dy}{dx}\right)^2 = 9(x+4)$

$= 9x + 36$

$L = \int_0^2 \sqrt{1 + (9x+36)}\,dx$

$= \int_0^2 \sqrt{9x+37}\,dx$

* if needed, can use u-sub to antidifferentiate

$= \frac{1}{9}\cdot\frac{2}{3}(9x+37)^{3/2}\Big]_0^2$

$\boxed{\frac{2}{27}(55)^{3/2} - \frac{2}{27}(37)^{3/2}}$