p. 527 # 12 - 22

12. $\int_{-\infty}^{\infty} (y^3 - 3y^2)\, dy$

$= \lim_{t \to -\infty} \int_t^0 y^3 - 3y^2 \, dy \quad + \quad \lim_{t \to \infty} \int_0^t y^3 - 3y^2 \, dy$

$= \frac{1}{4}y^4 - y^3 \Big]_t^0$

$= [0] - [\frac{1}{4}t^4 - t^3]$

$= -\frac{1}{4}(-\infty)^4 + (-\infty)^3$

DIVERGES

$= \frac{1}{4}y^4 - y^3 \Big]_0^t$

$= [\frac{1}{4}t^4 - t^3] - 0$

$= \frac{1}{4}\infty^4 - \infty^3$

DIVERGES

So integral DIVERGES

---

13. $\int_{-\infty}^{\infty} x e^{-x^2} \, dx$

$= \lim_{t \to -\infty} \int_t^0 x e^{-x^2} \, dx \quad + \quad \lim_{t \to \infty} \int_0^t x e^{-x^2}$

$u = -x^2$

$du = -2x \, dx$

$-\frac{1}{2} du = x \, dx$

$= -\frac{1}{2} \int e^u \, du$

$= -\frac{1}{2} e^{-x^2} \Big]_t^0$

$= [-\frac{1}{2}] - [-\frac{1}{2}e^{-t^2}]$

$= -\frac{1}{2} + \cancel{\frac{1}{2}e^{-\infty^2}}$ 0

$-\frac{1}{2} \int e^u \, du$

$-\frac{1}{2} e^{-x^2} \Big]_0^t$

$= [-\frac{1}{2}e^{-t^2}] - [-\frac{1}{2}]$

$= \cancel{-\frac{1}{2}e^{-\infty^2}}$ 0 $+ \frac{1}{2}$

$-\frac{1}{2} + \frac{1}{2} = 0$

converges

14. $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx = \lim_{t\to\infty} \int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

$u = -\sqrt{x}\,dx$

$du = -\frac{1}{2}x^{-1/2}\,dx$

$du = \frac{-1}{2\sqrt{x}}\,dx$

$-2\,du = \frac{-1}{\sqrt{x}}\,dx$

$= -2\int e^u\,du$

$= -2e^u = -2e^{-\sqrt{x}}\Big]_1^t$

$= \left[-2e^{-\sqrt{t}}\right] - \left[-2e^{-\sqrt{1}}\right] = \cancel{-2e^{-\sqrt{\infty}}}^{\,0} + \boxed{\frac{2}{e}}$ converges

---

15. $\int_0^\infty \sin^2\alpha\,d\alpha = \lim_{t\to\infty}\int_0^t \frac{1}{2} - \frac{1}{2}\cos 2\alpha\,d\alpha$

$= \frac{1}{2}\alpha - \frac{1}{4}\sin 2\alpha\Big]_0^t$

$= \left[\frac{1}{2}t - \frac{1}{4}\sin 2t\right] - \left[\cancel{\frac{1}{2}\cdot 0 - \frac{1}{4}\sin(2\cdot 0)}^{\,0}\right]$

$= \frac{1}{2}\infty - \frac{1}{4}\sin 2\infty$

diverges

---

16. $\int_{-\infty}^\infty \cos\pi t\,dt = \lim_{t\to-\infty}\int_t^0 \cos\pi t\,dt + \lim_{t\to\infty}\int_0^t \cos\pi t\,dt$

$= +\frac{1}{\pi}\sin\pi t\Big]_t^0 + \frac{1}{\pi}\sin\pi t\Big]_0^t$

$\left[\cancel{\frac{1}{\pi}\sin\pi 0}^{\,0}\right] - \left[\frac{1}{\pi}\sin\pi t\right] + \left[\frac{1}{\pi}\sin\pi t\right] - \left[\cancel{\frac{1}{\pi}\sin\pi\cdot 0}^{\,0}\right]$

$-\frac{1}{\pi}\sin(\pi\cdot\infty) + \left[\frac{1}{\pi}\sin(\pi\cdot\infty)\right]$

diverges

17. $\displaystyle\int_1^\infty \frac{1}{\cancel{x^2+x}} dx$ $= \displaystyle\lim_{t\to\infty} \int_1^t \frac{A}{x} + \frac{B}{x+1}\, dx$

$x(x+1)$

$$\frac{1}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$$1 = A(x+1) + B(x)$$

$$1 = Ax + A + Bx$$

$$1 = \underbrace{(A+B)}_{0}x + \underbrace{A}_{1} \longrightarrow \text{so } A=1 \;\&\; B=-1$$

$$\lim_{t\to\infty} \int_1^t \frac{1}{x} + \frac{-1}{x+1}\, dx = \ln|x| - \ln|x+1| \Big]_1^t$$

$$= \left[\ln|t| - \ln|t+1|\right] - \left[\overset{0}{\cancel{\ln}}|1| - \ln|2|\right]$$

$$= \underbrace{\cancel{\ln\infty}}_{\infty} - \underbrace{\cancel{\ln|\infty+1|}}_{\infty} + \ln|2| = \cancel{\infty - \infty} + \boxed{\ln 2}$$

converges

18. $\displaystyle\int_8^\infty \frac{dv}{\cancel{v^2-5v+4}}$ $= \displaystyle\lim_{t\to\infty} \int_8^{t} \frac{A}{v-4} + \frac{B}{v-1}\, dv$

$(v-4)(v-1)$

$$\frac{1}{(v-4)(v-1)} = \frac{A}{v-4} + \frac{B}{v-1}$$

$$1 = A(v-1) + B(v-4)$$

$$1 = Av - A + Bv - 4B$$

$$1 = \underbrace{(A+B)}_{0}v \underbrace{- A - 4B}_{1}$$

$A+B=0$

$+ \; -A-4B=1$

$-3B = 1$

$B = -1/3$

$A = 1/3$

$$\lim_{t\to\infty} \int_8^t \frac{\frac{1}{3}}{v-4} + \frac{-\frac{1}{3}}{v-1}\, dv = \frac{1}{3}\ln|v-4| + -\frac{1}{3}\ln|v-1| \Big]_8^t$$

$$= \left[\frac{1}{3}\ln|t-4| - \frac{1}{3}\ln|t-1|\right] - \left[+\frac{1}{3}\ln 4 - \frac{1}{3}\ln 7\right]$$

$$= \underbrace{\cancel{\frac{1}{3}\ln\infty}}_{0} - \underbrace{\cancel{\frac{1}{3}\ln\infty}}_{0} \boxed{-\frac{1}{3}\ln\frac{4}{7}}$$ converges

*note this is a POSITIVE #

19. $\int_{-\infty}^{0} z \cdot e^{2z}\, dz = \lim_{t \to -\infty} \int_{t}^{0} z \cdot e^{2z}\, dz$ $\quad u = z \quad v = \frac{1}{2}e^{2z}$ $\quad du = dz \quad dv = e^{2z}\, dz$

$= \lim_{t \to -\infty} z \cdot \frac{1}{2} e^{2z} - \int \frac{1}{2} e^{2z}\, dz$

$= \frac{z}{2} e^{2z} - \frac{1}{4} e^{2z} \Big]_{t}^{0}$

$= \left[0 - \frac{1}{4}e^{0}\right] - \left[\frac{t}{2}e^{2t} - \frac{1}{4}e^{2t}\right]$

$= -\frac{1}{4} - \cancel{\frac{\infty}{2} e^{2\cdot\infty}}^{0} - \cancel{\frac{1}{4} e^{2\cdot\infty}}^{0} = \boxed{-\frac{1}{4}}$ converges

* L'Hospitals

---

20. $\int_{2}^{\infty} y e^{-3y}\, dy = \lim_{t \to \infty} \int_{2}^{t} y e^{-3y}\, dy$ $\quad u = y \quad v = -\frac{1}{3}e^{-3y}$ $\quad du = dy \quad dv = e^{-3y}\, dy$

$= y \cdot -\frac{1}{3}e^{-3y} - \int -\frac{1}{3} e^{-3y}\, dy = -\frac{1}{3} y e^{-3y} + \int \frac{1}{3} e^{-3y}\, dy$

$= -\frac{1}{3} y e^{-3y} - \frac{1}{9} e^{-3y} \Big]_{2}^{t}$

$= \left[\cancel{-\frac{1}{3} t e^{-3t}}^{0} - \cancel{\frac{1}{9} e^{-3t}}^{0}\right] - \left[-\frac{1}{3} \cdot 2 e^{-3\cdot 2} - \frac{1}{9} e^{-3 \cdot 2}\right]$

$= \frac{2}{3} e^{-6} + \frac{1}{9} e^{-6} = \boxed{\frac{7}{9} e^{-6}}$ converges

* L'Hospitals

21. $\int_1^\infty \frac{\ln x}{x}\,dx = \lim_{t\to\infty} \int_1^t \frac{\ln x}{x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$

$= \int u\,du = \frac{1}{2}u^2\Big] = \frac{1}{2}(\ln x)^2\Big]_1^t$

$= \frac{1}{2}(\ln t)^2 - \cancel{\frac{1}{2}(\ln 1)}^{0} = \frac{1}{2}(\ln\infty)^2 = \infty$ diverges

---

22. $\int_{-\infty}^{\infty} x^3 e^{-x^4}\,dx$

$u = -x^4$
$du = -4x^3\,dx$
$-\frac{1}{4}\,du = x^3\,dx$

$= \lim_{t\to-\infty} \int_t^0 x^3 e^{-x^4}\,dx + \lim_{t\to\infty} \int_0^t x^3 e^{-x^4}\,dx$

$\lim_{t\to-\infty} -\frac{1}{4}\int_{-t^4}^{0} e^u\,du + -\frac{1}{4}\int_0^{-t^4} e^u\,du$

$-\frac{1}{4}e^u\Big]_{-t^4}^{0} + -\frac{1}{4}e^u\Big]_0^{-t^4}$

$\left[-\frac{1}{4}e^0\right] - \left[\cancel{-\frac{1}{4}e^{-t^4}}\right] + \left[-\frac{1}{4}e^{-t^4}\right] - \left[-\frac{1}{4}e^0\right]$

$-\frac{1}{4} + \cancel{\frac{1}{4}e^{-t^4}} + \cancel{-\frac{1}{4}e^{-t^4}} + \frac{1}{4}$