

$$1. \int \frac{e^x}{1+2e^x} dx$$

$$u = 1 + 2e^x$$

$$\frac{du}{dx} = 2e^x$$

$$du = 2e^x dx$$

$$\frac{1}{2} du = e^x dx$$

$$\int \frac{\frac{1}{2} du}{u} = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln|u| + C$$

$$= \frac{1}{2} \ln|1+2e^x| + C$$

$$2. \int \sec^2(2x) dx$$

$$u = 2x$$

$$\frac{du}{dx} = 2$$

$$du = 2 dx$$

$$\frac{1}{2} du = dx$$

$$\frac{1}{2} \int \sec^2 u du$$

$$= \frac{1}{2} \tan u + C$$

$$= \frac{1}{2} \tan(2x) + C$$

$$3. \int \sec^2(3x) e^{\tan(3x)} dx$$

$$u = \tan 3x$$

$$\frac{du}{dx} = \sec^2 3x \cdot 3$$

$$\frac{1}{3} du = \sec^2 3x dx$$

$$\int \frac{1}{3} du \cdot e^u = \frac{1}{3} \int e^u du$$

$$= \frac{1}{3} \cdot e^u + C$$

$$= \frac{1}{3} e^{\tan 3x} + C$$

$$4. \int \frac{x}{2x^2+1} dx$$

$$u = 2x^2 + 1$$

$$du = 4x dx$$

$$\frac{1}{4} du = x dx$$

$$\int \frac{\frac{1}{4} du}{u} = \frac{1}{4} \int \frac{1}{u} du$$

$$= \frac{1}{4} \ln|u| + C$$

$$= \frac{1}{4} \ln|2x^2+1| + C$$

$$5. \int e^x (2+e^x)^{1/2} dx$$

$$u = 2 + e^x$$

$$du = e^x dx$$

$$\int u^{1/2} du = \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3} (2+e^x)^{3/2} + C$$

$$6. \int x^2 \cos(x^3) dx$$

$$u = x^3$$

$$du = 3x^2 dx$$

$$\frac{1}{3} du = x^2 dx$$

$$\int \cos u \cdot \frac{1}{3} du = \frac{1}{3} \int \cos u du$$

$$= \frac{1}{3} \sin u + C$$

$$= \frac{1}{3} \sin(x^3) + C$$

$$7. \int \frac{\sec^2 x \, dx}{\sqrt{\tan x}}$$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\int \frac{du}{\sqrt{u}} = \int u^{-1/2} \, du$$

$$= 2u^{1/2} + C$$

$$= 2(\tan x)^{1/2} + C$$

$$8. \int \frac{\tan^{-1} x \, dx}{1+x^2}$$

$$u = \tan^{-1} x$$

$$du = \frac{1}{1+x^2} \, dx$$

$$\int u \, du = \frac{u^2}{2} + C$$

$$= \frac{1}{2}(\tan^{-1} x)^2 + C$$

$$9. \int \csc^2(3x+5) \, dx$$

$$u = 3x+5$$

$$du = 3 \, dx$$

$$\frac{1}{3} \, du = dx$$

$$\int \csc^2 u \cdot \frac{1}{3} \, du = \frac{1}{3} \int \csc^2 u \, du$$

$$= \frac{1}{3} \cdot -\cot u + C$$

$$= -\frac{1}{3} \cot(3x+5) + C$$