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3. $y = \sin x,\ 0 \le x \le \pi$

$$\left.\begin{aligned} \frac{dy}{dx} &= \cos x \\ \left(\frac{dy}{dx}\right)^2 &= \cos^2 x \end{aligned}\right\} \quad L = \int_0^{\pi} \sqrt{1 + \cos^2 x}\; dx$$

$$\boxed{\approx 3.8202}$$

4. $y = xe^{-x},\ 0 \le x \le 2$

$$\left.\begin{aligned} \frac{dy}{dx} &= x \cdot -e^{-x} + e^{-x} \cdot 1 \\ \left(\frac{dy}{dx}\right)^2 &= \left(e^{-x}(1-x)\right)^2 \end{aligned}\right\} \quad L = \int_0^{2} \sqrt{1 + (e^{-x}(1-x))^2}\; dx$$

$$\boxed{\approx 2.1024}$$

5. $x = \sqrt{y} - y,\ 1 \le y \le 4$

$$\left.\begin{aligned} \frac{dx}{dy} &= \frac{1}{2} y^{-1/2} - 1 \\ \left(\frac{dx}{dy}\right)^2 &= \left(\frac{1}{2\sqrt{y}} - 1\right)^2 \end{aligned}\right\} \quad L = \int_1^{4} \sqrt{1 + \left(\frac{1}{2\sqrt{y}} - 1\right)^2}\; dy$$

$$\boxed{\approx 3.6095}$$

6. $x = y^2 - 2y,\ 0 \le y \le 2$

$$\left.\begin{aligned} \frac{dx}{dy} &= 2y - 2 \\ \left(\frac{dx}{dy}\right)^2 &= (2y-2)^2 \end{aligned}\right\} \quad L = \int_0^{2} \sqrt{1 + (2y-2)^2}\; dy$$

$$\boxed{\approx 2.9579}$$

7. $y = 1 + 6x^{3/2}, \quad 0 \le x \le 1$

$\frac{dy}{dx} = 9x^{1/2}$

$\left(\frac{dy}{dx}\right)^2 = 81x$

$$L = \int_0^1 \sqrt{\underbrace{1 + 81x}_{u}}\ \overset{\frac{1}{81}du}{dx}$$

$u = 1 + 81x$

$du = 81\ dx$

$\frac{1}{81} du = dx$

$$= \frac{1}{81}\int_1^{82} u^{1/2}\, du = \frac{1}{81}\cdot\frac{2}{3}u^{3/2}\Big]_1^{82}$$

$$= \frac{2}{243}u^{3/2}\Big]_1^{82}$$

$$= \boxed{\frac{2}{243}(82)^{3/2} - \frac{2}{243}}$$

8. $y^2 = 4(x+4)^3, \quad 0 \le x \le 2, \quad y > 0$

so $y = 2(x+4)^{3/2}$

and $\frac{dy}{dx} = 3(x+4)^{1/2}$

then $\left(\frac{dy}{dx}\right)^2 = 9(x+4)$

$$L = \int_0^2 \sqrt{1 + 9x + 36}\ dx$$

$$= \int_0^2 \sqrt{37 + 9x} = \frac{1}{9}\cdot\frac{2}{3}(37+9x)^{3/2}\Big]_0^2$$

$$= \boxed{\frac{2}{27}(37+18)^{3/2} - \frac{2}{27}(37)^{3/2}}$$

9. $y = \frac{x^3}{3} + \frac{1}{4x}, \quad 1 \le x \le 2$

$\frac{dy}{dx} = x^2 - \frac{1}{4x^2}$

$\left(\frac{dy}{dx}\right)^2 = x^4 - \frac{1}{2} + \frac{1}{16x^4}$

$$L = \int_1^2 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16x^4}}\ dx = \int_1^2 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16x^4}}\ dx$$

$$= \int_1^2 \sqrt{\left(x^2 + \frac{1}{4x^2}\right)^2}\ dx = \int_1^2 x^2 + \frac{1}{4x^2}\ dx$$

$$= \frac{1}{3}x^3 - \frac{1}{4x}\Big]_1^2$$

$$= \left[\frac{1}{3}(2)^3 - \frac{1}{4(2)}\right] - \left[\frac{1}{3}(1)^3 - \frac{1}{4(1)}\right]$$

$$= \frac{8}{3} - \frac{1}{8} - \frac{1}{3} + \frac{1}{4} = \boxed{\frac{59}{24}}$$

10. $x = \frac{y^4}{8} + \frac{1}{4y^2}, \quad 1 \le y \le 2$

$$\frac{dx}{dy} = \frac{1}{2}y^3 - \frac{1}{2y^3}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y^6 - \frac{1}{2} + \frac{1}{4}y^{-6}$$

$$L = \int_1^2 \sqrt{1 + \frac{1}{4}y^6 - \frac{1}{2} + \frac{1}{4}y^{-6}}\, dy$$

$$= \int_1^2 \sqrt{\frac{1}{4}y^6 + \frac{1}{2} + \frac{1}{4}y^{-6}}\, dy$$

$$= \int_1^2 \sqrt{\left(\frac{1}{2}y^3 + \frac{1}{2}y^{-3}\right)^2}\, dy = \int_1^2 \frac{1}{2}y^3 + \frac{1}{2}y^{-3}\, dy$$

$$= \frac{1}{8}y^4 - \frac{1}{4}y^{-2}\Big]_1^2 = \left(\frac{1}{8}(2)^4 - \frac{1}{4}(2)^{-2}\right) - \left(\frac{1}{8} - \frac{1}{4}\right)$$

$$= 2 - \frac{1}{16} - \frac{1}{8} + \frac{1}{4} = \boxed{\frac{33}{16}}$$

11. $x = \frac{1}{3}\sqrt{y}\,(y-3), \quad 1 \le y \le 9$

$$x = \frac{1}{3}y^{3/2} - y^{1/2}$$

$$\frac{dx}{dy} = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4}y - \frac{1}{2} + \frac{1}{4}y^{-1}$$

$$L = \int_1^9 \sqrt{1 + \frac{1}{4}y - \frac{1}{2} + \frac{1}{4}y^{-1}}\, dy$$

$$= \int_1^9 \sqrt{\frac{1}{4}y + \frac{1}{2} + \frac{1}{4}y^{-1}}\, dy$$

$$= \int_1^9 \sqrt{\left(\frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\right)^2}\, dy = \int_1^9 \frac{1}{2}y^{1/2} + \frac{1}{2}y^{-1/2}\, dy$$

$$= \frac{1}{3}y^{3/2} + y^{1/2}\Big]_1^9$$

$$= \left[\frac{1}{3}(9)^{3/2} + (9)^{1/2}\right] - \left[\frac{1}{3}(1)^{3/2} + (1)^{1/2}\right]$$

$$= 9 + 3 - \frac{1}{3} - 1$$

$$= \boxed{\frac{32}{3}}$$