① $\int_1^{\sqrt{2}} 2x\,dx = x^2\Big]_1^{\sqrt{2}} = (\sqrt{2})^2 - (1)^2$

$= 2 - 1$

$= \boxed{1}$

no u-sub needed

---

② $\int_0^2 \sqrt{\underbrace{4x+1}_{u}}\,\overbrace{dx}^{\frac{1}{4}du}$

$u = 4x + 1$

$du = 4\,dx$

$\frac{1}{4}\,du = dx$

$\frac{1}{4}\int_1^9 u^{1/2}\,du = \frac{1}{4}\cdot\frac{2}{3}u^{3/2}\Big]_1^9$

$= \frac{1}{6}(9)^{3/2} - \frac{1}{6}(1)^{3/2}$

$= \frac{27}{6} - \frac{1}{6} = \frac{26}{6} = \boxed{\frac{13}{3}}$

---

③ $\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \tan^{-1}x\Big]_{-1}^{1}$

no u-sub needed

$= (\arctan 1) - (\arctan(-1))$

$= \pi/4 - (-\pi/4) = \boxed{\pi/2}$

---

④ $\int_0^1 \frac{1}{\underbrace{(2x+3)}_{u}^3}\,\overbrace{dx}^{\frac{1}{2}du}$

$u = 2x + 3$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$\frac{1}{2}\int_3^5 \frac{1}{u^3}\,du = \frac{1}{2}\cdot\frac{u^{-2}}{-2}\Big]_3^5$

$= \frac{1}{2}\left[\left(-\frac{1}{2}(5)^{-2}\right) - \left(-\frac{1}{2}(3)^{-2}\right)\right]$

$= \frac{1}{2}\left[-\frac{1}{50} + \frac{1}{18}\right]$

(5) $\int_e^{e^2} \frac{1}{x \ln x}\,dx$ ($\ln x = u$, du)

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int_1^2 \frac{1}{u}\,du = \ln|u|\Big]_1^2$$

$$= \ln 2 - \ln 1 = \boxed{\ln 2}$$

---

(6) $\int_2^{e+1} \frac{x^{u+1}}{(x-1)^2}\,dx$ ($x-1 = u$, du)

$u = x - 1$

$du = dx$

$$\int_1^e \frac{u+1}{u^2}\,du = \int_1^e \left(\frac{u}{u^2} + \frac{1}{u^2}\right)du = \int_1^e \left(\frac{1}{u} + u^{-2}\right)du$$

$$= \frac{u^{-1}}{-1} + \ln|u|\Big]_1^e = \left(-\frac{1}{e} + \ln e\right) - (-1 + \ln 1)$$

$$= \left(-\frac{1}{e} + 1\right) - (-1 + 0) = \boxed{2 - \frac{1}{e}}$$

---

(7) $\int_0^{\pi} \sin\left(\frac{x}{2}\right)dx$ ($\frac{x}{2} = u$, $2\,du$)

$u = \frac{x}{2}$

$du = \frac{1}{2}\,dx$

$2\,du = dx$

$$2\int_0^{\pi/2} \sin u\,du$$

$$= -2\cos u\Big]_0^{\pi/2}$$

$$= (-2\cos \pi/2) - (-2\cos 0)$$

$$= 0 + 2$$

$$= \boxed{2}$$

(8) $\int_{-\pi}^{\pi} x \sin(x^2)\, dx$ ($u = x^2$, $\frac{1}{2}du$)

$u = x^2$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$$\frac{1}{2}\int_{\pi^2}^{\pi^2} \sin u\, du = 0$$

---

(9) $\int_{-1}^{0} \frac{2}{6x-1}\, dx$ (u, $\frac{1}{3}du$)

$u = 6x - 1$
$du = 6\,dx$
$\frac{1}{3}du = 2\,dx$

$$\frac{1}{3}\int_{-7}^{-1} \frac{du}{u} = \frac{1}{3}\int_{-7}^{-1} \frac{1}{u}\, du = \left(\frac{1}{3}\ln|-1|\right) - \left(\frac{1}{3}\ln|-7|\right)$$

$$= \left(\frac{1}{3}\ln(1)\right) - \left(\frac{1}{3}\ln 7\right)$$

$$= 0 - \frac{1}{3}\ln 7 = -\frac{1}{3}\ln 7$$

---

(10) $\int_{1}^{5} \frac{1}{x}(\ln x)^{1/2}\, dx$ (u, du)

$u = \ln x$
$du = \frac{1}{x}dx$

$$\int_{0}^{\ln 5} u^{1/2}\, du = \frac{2}{3}u^{3/2}\Big]_{0}^{\ln 5}$$

$$= \left(\frac{2}{3}(\ln 5)^{3/2}\right) - \frac{2}{3}(0)^{3/2}$$

$$= \ln 5$$

(11) $\int_0^1 x e^{-x^2}\,dx$

u over $-x^2$; $-\frac{1}{2}\,du$ over $x\,dx$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}\,du = x\,dx$

$$-\frac{1}{2}\int_0^{-1} e^u\,du = \frac{1}{2}\int_{-1}^{0} e^u\,du$$

$$= \frac{1}{2}e^u\Big]_{-1}^{0} = \left(\frac{1}{2}e^0\right) - \left(\frac{1}{2}e^{-1}\right)$$

$$= \frac{1}{2} - \frac{1}{2e} = \boxed{\frac{1}{2}\left(1 - \frac{1}{e}\right)}$$

---

(12) $\int_0^2 (2 + x^2)\,dx = 2x + \frac{1}{3}x^3\Big]_0^2$

$$= \left(2(2) + \frac{1}{3}(2)^3\right) - 0$$

$$= 4 + \frac{8}{3} = \boxed{\frac{20}{3}}$$

no u-sub needed