10. $\int \frac{x+1}{(x^2+2x+7)^3}\,dx$ ($\frac{1}{2}du$; $u = x^2+2x+7$)

$u = x^2+2x+7$

$du = 2x+2\,dx$

$du = 2(x+1)\,dx$

$\frac{1}{2}du = x+1\,dx$

$\frac{1}{2}\int \frac{1}{u^3}\,du = \frac{1}{2}\int u^{-3}\,du$

$= \frac{1}{2}\cdot\frac{u^{-2}}{-2} + C$

$= -\frac{1}{4}(x^2+2x+7)^{-2} + C$

---

11. $\int \frac{x}{x^2-4}\,dx$ ($\frac{1}{2}du$; $u = x^2-4$)

$u = x^2-4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2-4| + C$

---

13. $\int \cos(3x)\,e^{\sin 3x}\,dx$ (u; $\frac{1}{3}du$)

$u = \sin 3x$

$du = 3\cos 3x\,dx$

$\frac{1}{3}du = \cos 3x\,dx$

$\frac{1}{3}\int e^u\,du$

$= \frac{1}{3}e^u + C$

$= \frac{1}{3}e^{\sin 3x} + C$

$\int \frac{1}{x\ln 3x}\,dx$ (du; u)

$u = \ln 3x$

$\frac{du}{dx} = \frac{1}{3x}\cdot 3$

$du = \frac{1}{x}\,dx$

$\int \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|\ln 3x| + C$

---

15. $\int \frac{\sin 3x}{1+\cos 3x}\,dx$ ($-\frac{1}{3}du$; $u = 1+\cos 3x$)

$u = 1+\cos 3x$

$du = -3\sin 3x\,dx$

$-\frac{1}{3}du = \sin 3x\,dx$

$-\frac{1}{3}\int \frac{1}{u}\,du$

$= -\frac{1}{3}\ln|u| + C$

$= -\frac{1}{3}\ln|1+\cos 3x| + C$

---

17. $\int \frac{1}{\sqrt{1-9x^2}}\,dx = \int \frac{1}{\sqrt{1-(3x)^2}}\,dx$ ($\frac{1}{3}du$; $u = 3x$)

$u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$\frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}\,du$

$= \frac{1}{3}\arcsin u + C$

$= \frac{1}{3}\arcsin(3x) + C$

18. $\int x \csc(3x^2)\cot(3x^2)\,dx$ (u = $3x^2$, $\frac{1}{6}\,du$)

$u = 3x^2$

$du = 6x\,dx$

$\frac{1}{6}\,du = x\,dx$

$\frac{1}{6}\int \csc u \cot u\,du$

$= \frac{1}{6}\cdot -\csc u + C$

$= -\frac{1}{6}\csc(3x^2) + C$

---

19. $\int \frac{1-e^{-x}}{x+e^{-x}}\,dx$ (du over the whole fraction, u = $x+e^{-x}$)

$u = x + e^{-x}$

$du = (1 - e^{-x})\,dx$

$= \frac{1}{u}\,du$

$= \ln|u| + C$

$= \ln|x + e^{-x}| + C$

---

20. $\int \frac{x^2-1}{x^2+1}\,dx = \int \frac{x^2 - 1 + 1 - 1}{x^2+1}\,dx = \int \frac{x^2+1}{x^2+1} + \frac{-2}{x^2+1}\,dx$

$= \int 1 - \frac{2}{x^2+1}\,dx$

$= x - 2\arctan x + C$

---

21. $\int \frac{x^5 - 35x}{x^2+6}\,dx$

$$\begin{array}{r} x^3 - 6x + \frac{x}{x^2+6} \\ x^2+6 \overline{)\,x^5 - 35x} \\ -x^5 \mp 6x^3 \\ \hline -6x^3 - 35 \\ +6x^3 + 36x \\ \hline x \end{array}$$

$= \int x^3 - 6x + \frac{x}{x^2+6}\,dx$ ($\frac{x}{x^2+6}$ = $\frac{1}{2}\,du$ over u)

$u = x^2 + 6$

$du = 2x\,dx$

$\frac{1}{2}\,du = x\,dx$

$\int x^3 - 6x\,dx + \frac{1}{2}\int \ln|u| + c$

$= \frac{1}{4}x^4 - \frac{6x^2}{2} + \frac{1}{2}\ln|u| + C$ $= \frac{1}{4}x^4 - 3x^2 + \frac{1}{2}\ln|x^2+6| + C$