TEST REVIEW (odd only)

True-False Quiz

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If f and g are continuous on $[a, b]$, then
$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

2. If f and g are continuous on $[a, b]$, then
$$\int_a^b [f(x)g(x)]\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$$

3. If f is continuous on $[a, b]$, then
$$\int_a^b 5f(x)\,dx = 5\int_a^b f(x)\,dx$$

4. If f is continuous on $[a, b]$, then
$$\int_a^b xf(x)\,dx = x\int_a^b f(x)\,dx$$

5. If f is continuous on $[a, b]$ and $f(x) \geqslant 0$, then
$$\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$$

6. If f' is continuous on $[1, 3]$, then $\int_1^3 f'(v)\,dv = f(3) - f(1)$.

7. If f and g are continuous and $f(x) \geqslant g(x)$ for $a \leqslant x \leqslant b$, then
$$\int_a^b f(x)\,dx \geqslant \int_a^b g(x)\,dx$$

8. If f and g are differentiable and $f(x) \geqslant g(x)$ for $a < x < b$, then $f'(x) \geqslant g'(x)$ for $a < x < b$.

9. $\displaystyle\int_{-1}^{1}\left(x^5 - 6x^9 + \frac{\sin x}{(1 + x^4)^2}\right)dx = 0$

10. $\displaystyle\int_{-5}^{5}(ax^2 + bx + c)\,dx = 2\int_0^5 (ax^2 + c)\,dx$

11. All continuous functions have derivatives.

12. All continuous functions have antiderivatives.

13. $\displaystyle\int_0^3 e^{x^2}\,dx = \int_0^5 e^{x^2}\,dx + \int_5^3 e^{x^2}\,dx$

14. If $\int_0^1 f(x)\,dx = 0$, then $f(x) = 0$ for $0 \leqslant x \leqslant 1$.

15. If f is continuous on $[a, b]$, then
$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x)$$

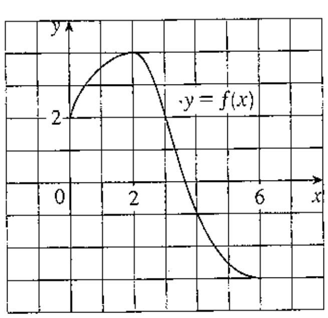
16. $\int_0^2 (x - x^3)\,dx$ represents the area under the curve $y = x - x^3$ from 0 to 2.

17. $\displaystyle\int_{-2}^{1}\frac{1}{x^4}\,dx = -\frac{3}{8}$

18. If f has a discontinuity at 0, then $\int_{-1}^{1} f(x)\,dx$ does not exist.

Exercises

1. Use the given graph of f to find the Riemann sum with six subintervals. Take the sample points to be (a) left endpoints and (b) midpoints. In each case draw a diagram and explain what the Riemann sum represents.



2. (a) Evaluate the Riemann sum for
$$f(x) = x^2 - x \qquad 0 \leqslant x \leqslant 2$$
with four subintervals, taking the sample points to be right endpoints. Explain, with the aid of a diagram, what the Riemann sum represents.
(b) Use the definition of a definite integral (with right endpoints) to calculate the value of the integral
$$\int_0^2 (x^2 - x)\,dx$$
(c) Use the Fundamental Theorem to check your answer to part (b).
(d) Draw a diagram to explain the geometric meaning of the integral in part (b).

Graphing calculator or computer required   CAS Computer algebra system required

3. Evaluate

$$\int_0^1 \left(x + \sqrt{1 - x^2}\right) dx$$

by interpreting it in terms of areas.

4. Express

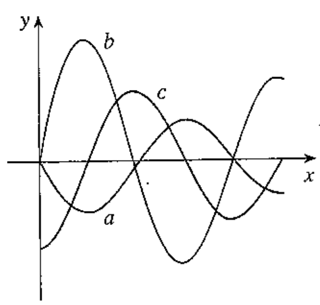
$$\lim_{n \to \infty} \sum_{i=1}^{n} \sin x_i \, \Delta x$$

as a definite integral on the interval $[0, \pi]$ and then evaluate the integral.

5. If $\int_0^6 f(x)\,dx = 10$ and $\int_0^4 f(x)\,dx = 7$, find $\int_4^6 f(x)\,dx$.

CAS 6. (a) Write $\int_1^5 (x + 2x^5)\,dx$ as a limit of Riemann sums, taking the sample points to be right endpoints. Use a computer algebra system to evaluate the sum and to compute the limit.
(b) Use the Fundamental Theorem to check your answer to part (a).

7. The following figure shows the graphs of f, f', and $\int_0^x f(t)\,dt$. Identify each graph, and explain your choices.



8. Evaluate:

(a) $\displaystyle\int_0^1 \frac{d}{dx}\left(e^{\arctan x}\right) dx$ (b) $\displaystyle\frac{d}{dx}\int_0^1 e^{\arctan x}\,dx$

(c) $\displaystyle\frac{d}{dx}\int_0^x e^{\arctan t}\,dt$

9–38 Evaluate the integral.

9. $\displaystyle\int_1^2 (8x^3 + 3x^2)\,dx$

10. $\displaystyle\int_0^T (x^4 - 8x + 7)\,dx$

11. $\displaystyle\int_0^1 (1 - x^9)\,dx$

12. $\displaystyle\int_0^1 (1 - x)^9\,dx$

13. $\displaystyle\int_1^9 \frac{\sqrt{u} - 2u^2}{u}\,du$

14. $\displaystyle\int_0^1 (\sqrt[4]{u} + 1)^2\,du$

15. $\displaystyle\int_0^1 y(y^2 + 1)^5\,dy$

16. $\displaystyle\int_0^2 y^2\sqrt{1 + y^3}\,dy$

17. $\displaystyle\int_1^5 \frac{dt}{(t - 4)^2}$

18. $\displaystyle\int_0^1 \sin(3\pi t)\,dt$

19. $\displaystyle\int_0^1 v^2\cos(v^3)\,dv$

20. $\displaystyle\int_{-1}^1 \frac{\sin x}{1 + x^2}\,dx$

21. $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{t^4 \tan t}{2 + \cos t}\,dt$

22. $\displaystyle\int_0^1 \frac{e^x}{1 + e^{2x}}\,dx$

23. $\displaystyle\int \left(\frac{1 - x}{x}\right)^2 dx$

24. $\displaystyle\int_1^{10} \frac{x}{x^2 - 4}\,dx$

25. $\displaystyle\int \frac{x + 2}{\sqrt{x^2 + 4x}}\,dx$

26. $\displaystyle\int \frac{\csc^2 x}{1 + \cot x}\,dx$

27. $\displaystyle\int \sin \pi t \cos \pi t\,dt$

28. $\displaystyle\int \sin x \cos(\cos x)\,dx$

29. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

30. $\displaystyle\int \frac{\cos(\ln x)}{x}\,dx$

31. $\displaystyle\int \tan x \ln(\cos x)\,dx$

32. $\displaystyle\int \frac{x}{\sqrt{1 - x^4}}\,dx$

33. $\displaystyle\int \frac{x^3}{1 + x^4}\,dx$

34. $\displaystyle\int \sinh(1 + 4x)\,dx$

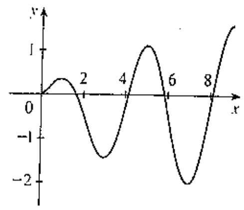
35. $\displaystyle\int \frac{\sec\theta \tan\theta}{1 + \sec\theta}\,d\theta$

36. $\displaystyle\int_0^{\pi/4} (1 + \tan t)^3 \sec^2 t\,dt$

37. $\displaystyle\int_0^3 |x - 4|\,dx$

38. $\displaystyle\int_0^4 |\sqrt{x} - 1|\,dx$

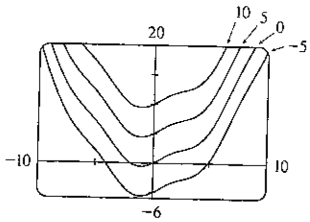
67. (a) Loc max at 1 and 5; loc min at 3 and 7
(b) $x = 9$
(c) $(\frac{1}{2}, 2)$, $(4, 6)$, $(8, 9)$
(d) See graph at right.



69. $\frac{1}{4}$ **77.** $f(x) = x^{3/2}$, $a = 9$
79. (b) Average expenditure over $[0, t]$; minimize average expenditure

EXERCISES 5.4 ■ PAGE 403

5. $\frac{1}{3}x^3 - (1/x) + C$ **7.** $\frac{1}{5}x^5 - \frac{1}{8}x^4 + \frac{1}{8}x^2 - 2x + C$
9. $\frac{2}{3}u^3 + \frac{9}{2}u^2 + 4u + C$ **11.** $\frac{1}{3}x^3 - 4\sqrt{x} + C$
13. $-\cos x + \cosh x + C$ **15.** $\frac{1}{2}\theta^2 + \csc\theta + C$
17. $\tan\alpha + C$
19. $\sin x + \frac{1}{4}x^2 + C$

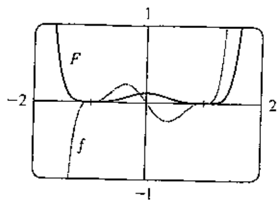


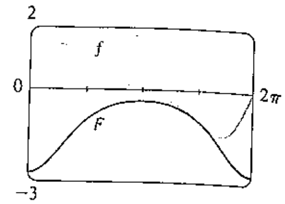
21. $-\frac{10}{3}$ **23.** $\frac{21}{5}$ **25.** -2 **27.** $5e^{\pi} + 1$ **29.** 36
31. $\frac{55}{63}$ **33.** $\frac{3}{4} - 2\ln 2$ **35.** $\dfrac{1}{11} + \dfrac{9}{\ln 10}$ **37.** $1 + \pi/4$
39. $\frac{256}{5}$ **41.** $\pi/3$ **43.** $\pi/6$ **45.** -3.5
47. ≈ 1.36 **49.** $\frac{4}{3}$
51. The increase in the child's weight (in pounds) between the ages of 5 and 10
53. Number of gallons of oil leaked in the first 2 hours
55. Increase in revenue when production is increased from 1000 to 5000 units
57. Newton-meters **59.** (a) $-\frac{3}{2}$ m (b) $\frac{41}{6}$ m
61. (a) $v(t) = \frac{1}{2}t^2 + 4t + 5$ m/s (b) $416\frac{2}{3}$ m
63. $46\frac{2}{3}$ kg **65.** 1.4 mi **67.** \$58,000
69. 5443 bacteria **71.** 4.75×10^5 megawatt-hours

EXERCISES 5.5 ■ PAGE 413

1. $-e^{-x} + C$ **3.** $\frac{2}{9}(x^3 + 1)^{3/2} + C$ **5.** $-\frac{1}{4}\cos^4\theta + C$
7. $-\frac{1}{2}\cos(x^2) + C$ **9.** $-\frac{1}{20}(1 - 2x)^{10} + C$
11. $\frac{1}{3}(2x + x^2)^{3/2} + C$ **13.** $-\frac{1}{3}\ln|5 - 3x| + C$
15. $-(1/\pi)\cos\pi t + C$ **17.** $\dfrac{1}{1 - e^u} + C$
19. $\frac{2}{3}\sqrt{3ax + bx^3} + C$ **21.** $\frac{1}{3}(\ln x)^3 + C$ **23.** $\frac{1}{4}\tan^4\theta + C$
25. $\frac{2}{3}(1 + e^x)^{3/2} + C$ **27.** $\frac{1}{15}(x^3 + 3x)^5 + C$
29. $-\dfrac{1}{\ln 5}\cos(5^t) + C$ **31.** $e^{\tan x} + C$ **33.** $-\dfrac{1}{\sin x} + C$

35. $-\frac{2}{3}(\cot x)^{3/2} + C$ **37.** $\frac{1}{3}\sinh^3 x + C$
39. $-\ln(1 + \cos^2 x) + C$ **41.** $\ln|\sin x| + C$
43. $\ln|\sin^{-1}x| + C$ **45.** $\tan^{-1}x + \frac{1}{2}\ln(1 + x^2) + C$
47. $\frac{1}{40}(2x + 5)^{10} - \frac{5}{36}(2x + 5)^9 + C$
49. $\frac{1}{8}(x^2 - 1)^4 + C$ **51.** $-e^{\cos x} + C$





53. $2/\pi$ **55.** $\frac{45}{28}$ **57.** 4 **59.** $e - \sqrt{e}$ **61.** 0
63. 3 **65.** $\frac{1}{3}(2\sqrt{2} - 1)a^3$ **67.** $\frac{16}{15}$ **69.** 2
71. $\ln(e + 1)$ **73.** $\frac{1}{6}$ **75.** $\sqrt{3} - \frac{1}{3}$ **77.** 6π
79. All three areas are equal. **81.** ≈ 4512 L
83. $\dfrac{5}{4\pi}\left(1 - \cos\dfrac{2\pi t}{5}\right)$ L **85.** 5 **91.** $\pi^2/4$

CHAPTER 5 REVIEW ■ PAGE 416

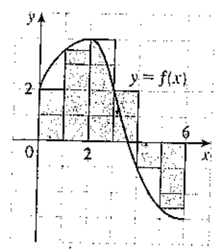
ANSWERS

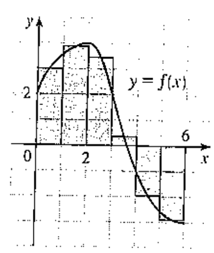
True-False Quiz

1. True **3.** True **5.** False **7.** True **9.** True
11. False **13.** True **15.** False **17.** False

Exercises

1. (a) 8 (b) 5.7





3. $\frac{1}{2} + \pi/4$ **5.** 3 **7.** f is c, f' is b, $\int_0^x f(t)\,dt$ is a
9. 37 **11.** $\frac{9}{10}$ **13.** -76 **15.** $\frac{21}{4}$ **17.** Does not exist
19. $\frac{1}{3}\sin 1$ **21.** 0 **23.** $-(1/x) - 2\ln|x| + x + C$
25. $\sqrt{x^2 + 4x} + C$ **27.** $\dfrac{1}{2\pi}\sin^2\pi t + C$
29. $2e^{\sqrt{x}} + C$ **31.** $-\frac{1}{2}[\ln(\cos x)]^2 + C$
33. $\frac{1}{4}\ln(1 + x^4) + C$ **35.** $\ln|1 + \sec\theta| + C$ **37.** $\frac{23}{3}$ 15
39. $2\sqrt{1 + \sin x} + C$ **41.** $\frac{64}{5}$ **43.** $F'(x) = x^2/(1 + x^3)$
45. $g'(x) = 4x^3\cos(x^8)$ **47.** $y' = (2e^x - e^{\sqrt{x}})/(2x)$
49. $4 \leqslant \int_1^3 \sqrt{x^2 + 3}\,dx \leqslant 4\sqrt{3}$ **55.** 0.280981
57. Number of barrels of oil consumed from Jan. 1, 2000, through Jan. 1, 2008