TEST REVIEW (odd only)

True-False Quiz

Determine whether the statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

1. If f and g are continuous on $[a, b]$, then

$$\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

T addition property of integrals

3. If f is continuous on $[a, b]$, then

$$\int_a^b 5f(x)\,dx = 5\int_a^b f(x)\,dx$$

T constant property of integrals

5. If f is continuous on $[a, b]$ and $f(x) \geqslant 0$, then

$$\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$$

F there is not an ~~property~~ exponent property

7. If f and g are continuous and $f(x) \geqslant g(x)$ for $a \leqslant x \leqslant b$, then

$$\int_a^b f(x)\,dx \geqslant \int_a^b g(x)\,dx$$

T if f(x) = + if g(x) = then area greater for f than g

9. $\displaystyle\int_{-1}^{1} \left(x^5 - 6x^9 + \frac{\sin x}{(1 + x^4)^2} \right) dx = 0$ * calculator only *

T SKIP

11. All continuous functions have derivatives.

F not differentiable at corner but continuous function

13. $\displaystyle\int_0^3 e^{x^2}\,dx = \int_0^5 e^{x^2}\,dx + \int_5^3 e^{x^2}\,dx$

T order of integration property $\int_0^3 = \int_0^5 - \int_3^5 = \int_0^5 + \int_5^3$

15. If f is continuous on $[a, b]$, then

$$\frac{d}{dx}\left(\int_a^b f(x)\,dx\right) = f(x)$$

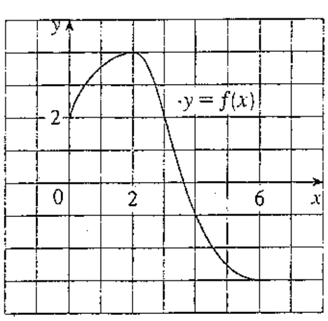
F

17. $\displaystyle\int_{-2}^{1} \frac{1}{x^4}\,dx = -\frac{3}{8}$ * see important comment on #17 in exercises below

F

Exercises

1. Use the given graph of f to find the Riemann sum with six subintervals. Take the sample points to be (a) left endpoints and (b) midpoints. In each case draw a diagram and explain what the Riemann sum represents.



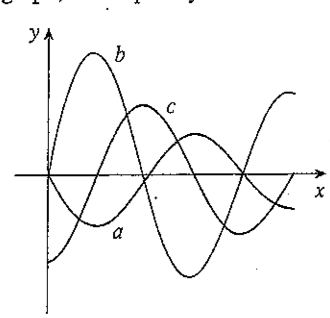
3. Evaluate

$$\int_0^1 \left(x + \sqrt{1 - x^2}\right) dx$$ SKIP

by interpreting it in terms of areas.

5. If $\int_0^6 f(x)\,dx = 10$ and $\int_0^4 f(x)\,dx = 7$, find $\int_4^6 f(x)\,dx$.

7. The following figure shows the graphs of f, f', and $\int_0^x f(t)\,dt$. Identify each graph, and explain your choices.



Graphing calculator or computer required

Exercises

LRAM 1. a. $\int_0^6 f(x)\,dx \approx \underline{1\cdot 2} + \underline{1\cdot 3.5} + \underline{1\cdot 4} + \underline{1\cdot 2} + \underline{1\cdot -1} + \underline{1\cdot -2.5}$

$\approx \boxed{8}$

RRAM ~~b. $\int_0^6 f(x)\,dx \approx 1\cdot 3.5 + 1\cdot 4 + 1\cdot 2 + 1\cdot -1 + 1\cdot -2.5 + 1\cdot -3$~~

MRAM b. $\int_0^6 f(x)\,dx \approx \underline{1\cdot 2.5} + \underline{1\cdot 3.75} + \underline{1\cdot 3.5} + \underline{1\cdot 0} + \underline{1\cdot -2} + \underline{1\cdot -2.5}$

$\approx \boxed{}$

3. SKIP

5. $\int_4^6 f(x)\,dx = \int_0^6 f(x)\,dx - \int_0^4 f(x)\,dx$

$= 10 - 7$

$= \boxed{3}$

7. * keep in mind that $\int_0^x f(t)\,dt = F(x)$ (the antider. of f)

$f \to c$

$f' \to b$

$\int_0^x f(t)\,dt \to a$

9. $= \frac{8x^4}{4} + \frac{3x^3}{3} \Big]_1^2 = \left[2(2)^4 + (2)^3\right] - \left[2(1)^4 + (1)^3\right]$

$= 32 + 8 - 2 \cancel{4} 1 = \cancel{4} \boxed{37}$

11. $= x - \frac{x^{10}}{10} \Big]_0^1 = \left(1 - \frac{1^{10}}{10}\right) - \left(0 - \frac{0^{10}}{10}\right)$

$= 1 - \frac{1}{10} = \boxed{\frac{9}{10}}$

13. rewrite:

$= \int_1^9 \left(\frac{\sqrt{u}}{u} - \frac{2u^2}{u}\right) du = \int_1^9 (u^{-1/2} - 2u)\, du$

$= 2u^{1/2} - \frac{2u^2}{2} \Big]_1^9 = \left(2(9)^{1/2} - (9)^2\right) - \left(2(1)^{1/2} - (1)^2\right)$

$= (2 \cdot 3 - 81) - (2 - 1)$

$= (6 - 81) - 1 = \boxed{-76}$

15. u-sub:

$\int_0^1 y(y^2+1)^5\, dy$

$u = y^2 + 1$

$du = 2y\, dy$

$\frac{1}{2} du = y\, dy$

$\frac{1}{2} \int_1^2 u^5\, du$

$= \frac{1}{2} \cdot \frac{u^6}{6} \Big]_1^2 = \frac{1}{12}(2)^6 - \frac{1}{12}(1)^6$

simplify

$= \frac{64}{12} - \frac{1}{12} = \boxed{\frac{63}{12}} = \boxed{\frac{21}{4}}$

17. u-sub:

$$\int_1^5 \frac{dt}{(t-4)^2}$$

$u = t-4$
$du = dt$

* To do the FTC p2... to find the area under the curve... f(x) must be continuous on interval. Note f(x) is not cont. at x=4 so... DNE

$$\int_{-3}^{1} u^{-2}\,du = -u^{-1}\Big]_{-3}^{1}$$

$$= \left(-(1)^{-1}\right) - \left(-(-3)^{-1}\right)$$

$$= (-1) - \left(-\tfrac{1}{3}\right) = -1 + \tfrac{1}{3} = \cancel{\boxed{-\tfrac{2}{3}}}$$

DNE

19. u-sub:

$$\int_0^1 v^2 \cos(v^3)\,dv$$

$u = v^3$
$du = 3v^2\,dv$
$\frac{1}{3}du = v^2\,dv$

$$\frac{1}{3}\int_0^1 \cos u\,du$$

$$= \frac{1}{3}\sin u\Big]_0^1 = \frac{1}{3}\sin 1 - \frac{1}{3}\sin 0$$

$$= \frac{1}{3}\cdot\sin 1 - \frac{1}{3}\cdot 0 = \boxed{\frac{1}{3}\sin 1}$$

$=$

23. rewrite:

$$\int \left(\frac{1}{x}-1\right)^2 dx = \int \left(\frac{1}{x}-1\right)\left(\frac{1}{x}-1\right) dx = \int \left(\frac{1}{x^2} - \frac{2}{x} + 1\right) dx$$

$$= \int (x^{-2} - 2x^{-1} + 1)\,dx = \boxed{-x^{-1} - 2\ln|x| + x + C}$$

25. u-sub:

$$\int \frac{x+2}{\sqrt{x^2+4x}}\,dx$$

$u = x^2 + 4x$

$du = 2x+4 \quad dx$

$du = 2(x+2) \quad dx$

$\frac{1}{2}\,du = (x+2)\,dx$

$$\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du$$

$$= \frac{1}{2}\int u^{-1/2}\,du = u^{1/2} + c$$

$$= \boxed{(x^2+4x)^{1/2} + c}$$

27. u-sub:

$$\int \sin\pi t \cos\pi t \; dt$$

$u = \sin\pi t$

$du = \pi\cos\pi t \; dt$

$\frac{1}{\pi}\,du = \cos\pi t \; dt$

$$\frac{1}{\pi}\int u\;du = \frac{1}{\pi}\cdot\frac{1}{2}u^2 + c$$

$$= \boxed{\frac{1}{2\pi}(\sin\pi t)^2 + c}$$

$$\cancel{= -\frac{1}{\pi}\cos u + c = -\frac{1}{\pi}\cos(\sin\pi t) + c}$$

29. u-sub:

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\;dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$2\int e^u\;du$$

$$= 2e^u + c = \boxed{2e^{\sqrt{x}} + c}$$

31. u-sub:

$$\int \tan x \ln(\cos x)\, dx$$

$u = \ln \cos x \, dx$

$du = \frac{1}{\cos x} \cdot -\sin x \, dx$

$$-\int u \, du$$

$-du = \frac{\sin x}{\cos x} dx$

$-du = \tan x \, dx$

$$= -\frac{1}{2}u^2 + c$$

$$= \boxed{-\frac{1}{2}(\ln \cos x)^2 + c}$$

33. u-sub:

$$\int \frac{x^3}{1+x^4} dx$$

$u = 1 + x^4$

$du = 4x^3 \, dx$

$\frac{1}{4} du = x^3 \, dx$

$$\frac{1}{4}\int \frac{1}{u} du$$

$$= \frac{1}{4}\ln|u| + c = \boxed{\frac{1}{4}\ln|1+x^4| + c}$$

35. u-sub:

$$\int \frac{\sec\theta \tan\theta}{1+\sec\theta} d\theta$$

$u = 1 + \sec\theta$

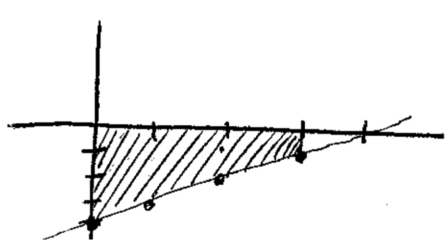
$du = \sec\theta \tan\theta \, d\theta$

$$\int \frac{1}{u} du$$

$$= \ln|u| + c = \boxed{\ln|1+\sec\theta| + c}$$

37. GRAPH

$$\int_0^3 |x-4| \, dx$$



all positive area!

$A_T = \frac{1}{2}h(b_1+b_2)$

$= \frac{1}{2}(3)(-4-1)$

$= \left|-\frac{15}{2}\right| \to \boxed{\frac{15}{2}}$