

p. 651

Find eq. of tangent at given pt.

3. $x = 1 + 4t - t^2$, $y = 2 - t^3$; $t = 1$

need POINT: $(1 + 4 \cdot 1 - 1^2, 2 - 1^3) = (4, 1)$

SLOPE: $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-3t^2}{4-2t}$

$\left. \frac{dy}{dx} \right|_{t=1} = \frac{-3(1)^2}{4-2(1)} = \frac{-3}{2}$

tangent:

$y - 1 = -\frac{3}{2}(x - 4)$

4. $x = t - t^{-1}$, $y = 1 + t^2$; $t = 1$

need POINT: $(1 - 1^{-1}, 1 + 1^2) = (0, 2)$

SLOPE: $\frac{dy}{dx} = \frac{2t}{1 + \frac{1}{t^2}}$

$\left. \frac{dy}{dx} \right|_{t=1} = \frac{2 \cdot 1}{1 + \frac{1}{1^2}} = \frac{2}{2} = 1$

tangent:

$y - 2 = 1(x - 0)$

5. $x = t \cos t$, $y = t \sin t$; $t = \pi$

need POINT: $(\pi \cos \pi, \pi \sin \pi) = (-\pi, 0)$

SLOPE: $\frac{dy}{dx} = \frac{t \cos t + \sin t}{-t \sin t + \cos t}$

$\left. \frac{dy}{dx} \right|_{t=\pi} = \frac{\pi \cos \pi + \sin \pi}{-\pi \sin \pi + \cos \pi} = \frac{-\pi}{-1} = \pi$

tangent:

$y - 0 = \pi(x + \pi)$

Find equation of tangent by:
 a - w/o eliminating parameter
 b - w/ eliminating parameter first

7. $x = 1 + \ln t$, $y = t^2 + 2$; (1, 3)

a. **need** POINT: $(1, 3)$

SLOPE: $\frac{dy}{dx} = \frac{2t}{1/t} = 2t^2$

$\frac{dy}{dx} \Big|_{t=1} = 2(1)^2 = 2$

tangent: $y - 3 = 2(x - 1)$

OR

$1 = 1 + \ln t$
 $0 = \ln t$
 $e^0 = t$
 $t = 1$

$3 = t^2 + 2$
 $1 = t^2$
 $t = \pm 1$

b. if $y = t^2 + 2$ and $t = e^{x-1}$, then:
 $y = (e^{x-1})^2 + 2$

need POINT: $(1, 3)$

SLOPE: $\frac{dy}{dx} = 2(e^{x-1}) \cdot e^{x-1}$

$\frac{dy}{dx} \Big|_{x=1} = 2(e^{1-1}) \cdot e^{1-1} = 2$

tangent: $y - 3 = 2(x - 1)$

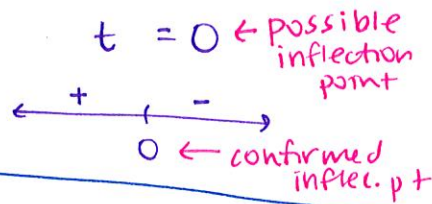
Find dy/dx and d^2y/dx^2 . For which values of t concave up?

11. $x = t^2 + 1, y = t^2 + t$

$$\frac{dy}{dx} = \frac{2t+1}{2t}$$

$$\frac{d^2y}{dx^2} = \frac{d/dt(dy/dx)}{dx/dt} = \frac{2t(2) - (2t+1)^2}{(2t)^2} = \frac{4t - 4t^2 - 4t - 1}{4t^2} = \frac{-4t^2 - 1}{4t^2} = -\frac{1}{4t^3} = \frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{4t^3} = 0 / = \text{DNE}$$



concave up: $(-\infty, 0)$

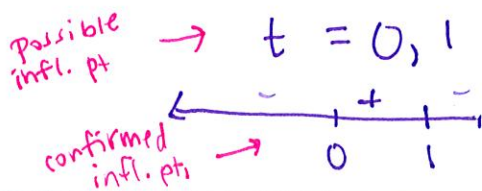
12. $x = t^3 + 1, y = t^2 - t$

$$\frac{dy}{dx} = \frac{2t-1}{3t^2}$$

$$\frac{d^2y}{dx^2} = \frac{(3t^2)(2) - (2t-1)(6t)}{9t^4} = \frac{6t^2 - 12t^2 + 6t}{27t^6} = \frac{-6t^2 + 6t}{27t^6}$$

$$\frac{d^2y}{dx^2} = \frac{-6t^2 + 6t}{27t^6} = 0 / = \text{DNE}$$

$$\frac{-6t^2 + 6t}{27t^6} = \frac{d^2y}{dx^2}$$



concave up: $(0, 1)$

Find pts on curve where tan is horizontal or vertical.

17. $x = t^3 - 3t$, $y = t^2 - 3$

horizontal:

$$\frac{dy}{dx} = \frac{2t}{3t^2 - 3} = 0$$

$$\left. \begin{array}{l} 2t = 0 \\ t = 0/2 \\ t = 0 \end{array} \right\} \text{point:}$$

$(0^3 - 3(0), 0^2 - 3)$
 $(0, -3)$

vertical:

$$\frac{dy}{dx} = \frac{2t}{3t^2 - 3} = 0$$

$$\left. \begin{array}{l} 3t^2 - 3 = 0 \\ 3t^2 = 3 \\ t^2 = 1 \\ t = \pm 1 \end{array} \right\} \text{points:}$$

$(-2, -2)$ AND
 $(2, -2)$

18. $x = t^3 - 3t$, $y = t^3 - 3t^2$

horizontal:

$$\frac{dy}{dx} = \frac{3t^2 - 6t}{3t^2 - 3} = 0$$

$$\left. \begin{array}{l} 3t^2 - 6t = 0 \\ 3t(t - 2) = 0 \\ t = 0, 2 \end{array} \right\} \text{points:}$$

$(0, 0)$ AND
 $(2, -4)$

vertical:

$$\frac{dy}{dx} = \frac{3t^2 - 6t}{3t^2 - 3} = 0$$

$$\left. \begin{array}{l} 3t^2 - 3 = 0 \\ 3(t^2 - 1) = 0 \\ 3(t+1)(t-1) = 0 \\ t = \pm 1 \end{array} \right\} \text{points:}$$

$(-2, -2)$
 $(2, -4)$