p. 651

Find eq. of tangent at given pt.

3. $x = 1 + 4t - t^2, \quad y = 2 - t^3; \quad t = 1$

need
POINT: $(1 + 4\cdot 1 - 1^2,\ 2 - 1^3) = \boxed{(4, 1)}$

SLOPE: $dy/dx = \dfrac{dy/dt}{dx/dt} = \dfrac{-3t^2}{4 - 2t}$

$dy/dx\Big|_{t=1} = \dfrac{-3(1)^2}{4 - 2(1)} = \boxed{\dfrac{-3}{2}}$

tangent: $\boxed{y - 1 = -\frac{3}{2}(x - 4)}$

4. $x = t - t^{-1}, \quad y = 1 + t^2; \quad t = 1$

need
POINT: $(1 - 1^{-1},\ 1 + 1^2) = \boxed{(0, 2)}$

SLOPE: $dy/dx = \dfrac{2t}{1 + 1/t^2}$

$dy/dx\Big|_{t=1} = \dfrac{2\cdot 1}{1 + 1/1^2} = \dfrac{2}{2} = \boxed{1}$

tangent: $\boxed{y - 2 = 1(x - 0)}$

5. $x = t\cos t, \quad y = t\sin t; \quad t = \pi$

need
POINT: $(\pi\cos\pi,\ \pi\sin\pi) = \boxed{(-\pi, 0)}$

SLOPE: $dy/dx = \dfrac{t\cos t + \sin t}{-t\sin t + \cos t}$

$dy/dx\Big|_{t=\pi} = \dfrac{\pi\cos\pi + \sin\pi}{-\pi\sin\pi + \cos\pi} = \dfrac{-\pi}{-1} = \boxed{\pi}$

tangent: $\boxed{y - 0 = \pi(x + \pi)}$

Find equation of tangent by:

a - w/o eliminating parameter

b - w/ eliminating parameter first

7. $x = 1 + \ln t, \quad y = t^2 + 2; \quad (1,3)$

a. need

POINT: $\boxed{(1,3)}$

SLOPE: $dy/dx = \dfrac{2t}{1/t} = 2t^2$

$dy/dx\Big|_{t=1} = 2(1)^2 = \boxed{2}$

tangent: $\boxed{y-3 = 2(x-1)}$

$1 = 1 + \ln t$

$0 = \ln t$

$e^0 = t$

$\boxed{t = 1}$

OR

$3 = t^2 + 2$

$1 = t^2$

$t = \pm 1$

b. if $y = t^2 + 2$ and $t = e^{x-1}$, then:

$y = (e^{x-1})^2 + 2$

need

POINT: $\boxed{(1,3)}$

SLOPE: $dy/dx = 2(e^{x-1}) \cdot e^{x-1}$

$dy/dx\Big|_{x=1} = 2(e^{1-1}) \cdot e^{1-1} = \boxed{2}$

tangent: $\boxed{y-3 = 2(x-1)}$

Find dy/dx and d^2y/dx^2. For which values of t concave up?

11. $x = t^2 + 1$, $y = t^2 + t$

$$\boxed{\frac{dy}{dx} = \frac{2t+1}{2t}}$$

$$\frac{d^2y}{dx^2} = \frac{d/dt\,(dy/dx)}{dx/dt} = \frac{\frac{2t(2) - (2t+1)2}{(2t)^2}}{2t} = \cancel{\frac{4t - 4t^2 - 4t - 1}{8t^3}}$$

$$= \frac{+\cancel{4t} - \cancel{4t} - 2}{8t^3} = \frac{-2}{8t^3} = \boxed{\frac{-1}{4t^3} = \frac{d^2y}{dx^2}}$$

$$\frac{d^2y}{dx^2} = \frac{-1}{4t^3} = 0 \;/ = \text{DNE}$$

$t = 0$ ← possible inflection point

+ | −

0 ← confirmed inflec. pt

$\boxed{\text{concave up! } (-\infty, 0)}$

12. $x = t^3 + 1$, $y = t^2 - t$

$$\boxed{\frac{dy}{dx} = \frac{2t-1}{3t^2}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{(3t^2)(2) - (2t-1)(6t)}{9t^4}}{3t^2} = \frac{6t^2 - 12t^2 + 6t}{27t^6}$$

$$= \boxed{\frac{-6t^2 + 6t}{27t^2} = \frac{d^2y}{dx^2}}$$

$$\frac{d^2y}{dx^2} = \frac{-6t^2 + 6t}{27t^6} = 0 \;/ = \text{DNE}$$

possible infl. pt → $t = 0, 1$

− | + | −

confirmed infl. pt → 0 1

$\boxed{\text{Concave up: } (0, 1)}$

Find pts on curve where tan is horizontal or vertical.

17. $x = t^3 - 3t$, $y = t^2 - 3$

horizontal:

$$\frac{dy}{dx} = \frac{2t}{3t^2-3} = 0$$

$2t = 0$

$t = 0/2$

$t = 0$

point:

$(0^3 - 3(0), 0^2 - 3)$

$\boxed{(0, -3)}$

vertical:

$$\frac{dy}{dx} = \frac{2t}{3t^2-3} = 0$$

$3t^2 - 3 = 0$

$3t^2 = 3$

$t^2 = 1$

$t = \pm 1$

points:

$\boxed{(-2, -2) \text{ AND } (2, -2)}$

---

18. $x = t^3 - 3t$, $y = t^3 - 3t^2$

horizontal:

$$\frac{dy}{dx} = \frac{3t^2 - 6t}{3t^2 - 3} = 0$$

$3t^2 - 6t = 0$

$3t(t-2) = 0$

$t = 0, 2$

points:

$\boxed{(0, 0) \text{ AND } (2, -4)}$

vertical:

$$\frac{dy}{dx} = \frac{3t^2 - 6t}{3t^2 - 3} = 0$$

$3t^2 - 3 = 0$

$3(t^2 - 1) = 0$

$3(t+1)(t-1) = 0$

$t = \pm 1$

points:

$\boxed{(-2, -2) \; (2, -4)}$