37. $x = t + e^{-t}, \quad y = t - e^{-t}, \quad 0 \le t \le 2$

$$\frac{dx}{dt} = 1 - e^{-t} \qquad \frac{dy}{dt} = 1 + e^{-t}$$

$$\left(\frac{dx}{dt}\right)^2 = (1-e^{-t})^2 \qquad \left(\frac{dy}{dt}\right)^2 = (1+e^{-t})^2$$

$$\boxed{L = \int_0^2 \sqrt{(1-e^{-t})^2 + (1+e^{-t})^2}\, dt \approx 3.1416}$$

38. $x = t^2 - t, \quad y = t^4, \quad 1 \le t \le 4$

$$\frac{dx}{dt} = 2t - 1 \qquad \frac{dy}{dt} = 4t^3$$

$$\left(\frac{dx}{dt}\right)^2 = 4t^2 - 4t + 1 \qquad \left(\frac{dy}{dt}\right)^2 = 16t^6$$

$$\boxed{L = \int_1^4 \sqrt{4t^2 - 4t + 1 + 16t^6}\, dt \approx 255.3756}$$

39. $x = t - 2\sin t, \quad y = 1 - 2\cos t, \quad 0 \le t \le 4\pi$

$$\frac{dx}{dt} = 1 - 2\cos t \qquad \frac{dy}{dt} = 2\sin t$$

$$\left(\frac{dx}{dt}\right)^2 = 1 - 4\cos t + 4\cos^2 t \qquad \left(\frac{dy}{dt}\right)^2 = 4\sin^2 t$$

$$\boxed{L = \int_0^{4\pi} \sqrt{1 - 4\cos t + 4\cos^2 t + 4\sin^2 t}\, dt \approx 26.7298}$$

41. $x = 1+3t^2, \quad y = 4+2t^3, \quad 0 \le t \le 1$

$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6t^2$

$\left(\frac{dx}{dt}\right)^2 = 36t^2 \qquad \left(\frac{dy}{dt}\right)^2 = 36t^4$

$L = \int_0^1 \sqrt{36t^2+36t^4}\, dt = \int_0^1 \sqrt{36t^2(1+t^2)}\, dt$

$= \int_0^1 6t\sqrt{1+t^2}\, dt$

$u = 1+t^2$
$du = 2t\ dt$
$3du = 6t\ dt$

$3\int_1^2 u^{1/2}\, du$

* note bound changes

$= 3\cdot\frac{2}{3}u^{3/2}\Big]_1^2 = 2u^{3/2}\Big]_1^2 = 2(2)^{3/2} - 2(1)^{3/2}$

$= \boxed{2(2^{3/2}-1)}$

42. $x = e^t + e^{-t}, \quad y = 5-2t,$
$0 \le t \le 3$

$\frac{dx}{dt} = e^t - e^{-t} \qquad \frac{dy}{dt} = -2$

$\left(\frac{dx}{dt}\right)^2 = e^{2t} - 2 + e^{-2t} \qquad \left(\frac{dy}{dt}\right)^2 = 4$

$L = \int_0^3 \sqrt{e^{2t}-2+e^{-2t}+4}\, dt = \int_0^3 \sqrt{e^{2t}+2+e^{-2t}}\, dt$

$= \int_0^3 \sqrt{(e^t+e^{-t})^2}\, dt = \int_0^3 e^t + e^{-t}\, dt$

$= e^t - e^{-t}\Big]_0^3 = \left[e^3 - e^{-3}\right] - \left[e^0 - e^{-0}\right]$

$= e^3 - e^{-3} \cancel{-1+1} = \boxed{e^3 - e^{-3}}$

Find the tan. line to:

$x = t^5 - 4t^3, \quad y = t^2; \quad$ at $(0,4)$

need: POINT: $\boxed{(0,4)}$

SLOPE: $\frac{dy}{dx} = \frac{2t}{5t^4 - 12t^2}$

$\left.\frac{dy}{dx}\right|_{t=2} \frac{2(2)}{5(2)^4 - 12(2)^2} = \boxed{\frac{1}{8}}$ $\quad \left.\frac{dy}{dx}\right|_{t=-2} = \frac{2(-2)}{5(-2)^4 - 12(-2)^2} = \boxed{-\frac{1}{8}}$

$0 = t^5 - 4t^3$

$0 = t^3(t^2 - 4)$ AND

$0 = t^3(t+2)(t-2)$

$t = 0, \pm 2$

$4 = t^2$

$t = \pm 2$

so...
$t = \pm 2$
only

tangents:

$$\boxed{\begin{aligned} y - 4 &= \tfrac{1}{8}(x - 0) \\ y - 4 &= -\tfrac{1}{8}(x - 0) \end{aligned}}$$

Determine pts where tan will be horizontal & vertical.

$x = t^3 - 3t, \quad y = 3t^2 - 9$

horizontal: $\frac{dy}{dx} = \frac{6t}{3t^2 - 3}$ $= 0$ & $\neq 0$ $\Rightarrow$ $6t = 0$, $t = 0$ $\}$ $\boxed{\text{point: } (0,-9) \text{ horizontal tangent}}$

vertical: $\frac{dy}{dx} = \frac{6t}{3t^2 - 3}$ $\neq 0$ & $= 0$ $\Rightarrow$

$3t^2 - 3 = 0$

$3(t^2 - 1) = 0$

$3(t+1)(t-1) = 0$

$t = \pm 1$

$\}$ $\boxed{\text{points: } (-2,-6) \quad (2,-6) \text{ vertical tangent}}$

Find the second derivative

$$x = t^5 - 4t^3, \qquad y = t^2$$

$$\frac{dy}{dx} = \frac{2t}{5t^4 - 12t^2}$$

$$\frac{d^2y}{dx^2} = \frac{\dfrac{(5t^4-12t^2)(2) - (2t)(20t^3-24t)}{(5t^4-12t^2)^2}}{5t^4-12t^2}$$

$$= \frac{10t^4 - 24t^2 - 40t^4 + 48t^2}{(5t^4-12t^2)^3}$$

$$= \boxed{\frac{-30t^4 + 24t^2}{(5t^4-12t^2)^3}}$$