1. (a) What is an alternating series?
   (b) Under what conditions does an alternating series converge?
   (c) If these conditions are satisfied, what can you say about the remainder after n terms?

2–20 Test the series for convergence or divergence.

2. $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \frac{2}{11} - \cdots$

3. $-\frac{2}{5} + \frac{4}{6} - \frac{6}{7} + \frac{8}{8} - \frac{10}{9} + \cdots$

4. $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} - \cdots$

5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$

7. $\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

8. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^3+2}}$

9. $\sum_{n=1}^{\infty} (-1)^n e^{-n}$

10. $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{2n+3}$

1a. alternating series - a series whose terms are alternately positive and negative

b. alt. ~~[illegible]~~ series converges when -

1) $a_{n+1} \leq a_n$ for all n

2) $\lim_{n \to \infty} a_n = 0$

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2. $\frac{2}{3} - \frac{2}{5} + \frac{2}{7} - \frac{2}{9} + \frac{2}{11} - \ldots = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \boxed{\frac{2}{2n+1}} \rightarrow a_n$

converges if: ① $\frac{2}{2(n+1)+1} \leq \frac{2}{2n+1}$

$$\frac{2}{2n+3} \leq \frac{2}{2n+1} \checkmark$$

② $\lim_{n \to \infty} \frac{2}{2n+1} = \frac{2}{2\infty+1} = 0 \checkmark$

so series converges by alt. series test

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3. $-\frac{2}{5} + \frac{4}{6} - \frac{6}{7} + \frac{8}{8} - \frac{10}{9} + \ldots = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{2n}{n+4}$

converges if: ① $\frac{2(n+1)}{(n+1)+4} \leq \frac{2n}{n+4}$

$$\frac{2n+2}{n+5} \leq \frac{2n}{n+4} \quad X$$

② $\lim_{n \to \infty} \frac{2n}{n+4} \rightarrow \frac{2}{1} = 2 \neq 0 \quad X$

Series DIVERGES by test for divergence

4. $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{6}} \cancel{\pm} \ldots = \sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{1}{\sqrt{n+1}}$

converges if: ① $\frac{1}{\sqrt{n+1+1}} \leq \frac{1}{\sqrt{n+1}}$

$\frac{1}{\sqrt{n+2}} \leq \frac{1}{\sqrt{n+1}}$ ✓

② $\lim_{n\to\infty} \frac{1}{\sqrt{n+1}} = \frac{1}{\infty} = 0$ ✓

series converges by Alt. series test

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5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

converges if: ① $\frac{1}{2(n+1)+1} \leq \frac{1}{2n+1}$

$\frac{1}{2n+3} \leq \frac{1}{2n+1}$ ✓

② $\lim_{n\to\infty} \frac{1}{2n+1} = \frac{1}{\infty} = 0$ ✓

series converges by Alt. series test

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6. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$

converges if: ① $\frac{1}{\ln(n+1+4)} \leq \frac{1}{\ln(n+4)}$

$\frac{1}{\ln(n+5)} \leq \frac{1}{\ln(n+4)}$ ✓

② $\lim_{n\to\infty} \frac{1}{\ln(n+4)} = \frac{1}{\infty} = 0$ ✓

Series converges by Alt. series test

7. $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{3n-1}{2n+1}$

converges if: ① $\frac{3(n+1)-1}{2(n+1)+1} \leq \frac{3n-1}{2n+1}$

$$\frac{3n+2}{2n+3} \leq \frac{3n-1}{2n+1}$$

② $\lim_{n\to\infty} \frac{3n-1}{2n+1} \to \frac{3}{2} \neq 0$

series diverges by Test for divergence

8. $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n}{\sqrt{n^3+2}}$

converges if: ① $\frac{n+1}{\sqrt{(n+1)^3+2}} \leq \frac{n}{\sqrt{n^3+2}}$ for $n \geq 2$

② $\lim_{n\to\infty} \frac{n}{\sqrt{n^3+2}} \to \frac{1}{\frac{1}{2}(n^3+2)^{-1/2}\cdot 3n^2} = \cancel{\frac{2\sqrt{n^3+2}}{3n^2}} \to$

$$\to \cancel{\frac{2\cdot\frac{1}{2}(n^3+2)^{-1/2}\cdot 3n^2}{6n}} = \cancel{\frac{3n}{2\sqrt{n^3+2}}} = 0$$

series converges by Alt. Test

9. $\sum_{n=1}^{\infty} (-1)^n e^{-n}$

converges if: ① $e^{-(n+1)} \leq e^{-n}$

$$\frac{1}{e^{n+1}} \leq \frac{1}{e^n}$$

② $\lim_{n\to\infty} e^{-n} = \frac{1}{e^n} = 0$

series converges by alt. series test

10. $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{\sqrt{n}}{2n+3}$

converges if:

(1) $\frac{\sqrt{n+1}}{2(n+1)+3} \leq \frac{\sqrt{n}}{2n+3}$ ✓

(2) $\lim_{n\to\infty} \frac{\sqrt{n}}{2n+3} \to \frac{\frac{1}{2}n^{-1/2}}{2} = \frac{1}{4\sqrt{n}} = \frac{1}{4\sqrt{\infty}} = 0$ ✓

so series converges by alt. series test