Vectors

1. If vector **v** has initial point (0, -3) and terminal point (6, 10),

a. put **v** in component form

$\langle 6, 13 \rangle$

b. Find $\|v\|$

$\sqrt{6^2+13^2} = \sqrt{36+169} = \sqrt{205}$

c. Write **v** in unit vector form.

$\vec{v} = 6\vec{i} + 13\vec{j}$

2. If $\mathbf{v}=\langle 2,3 \rangle$, **w** has an initial point (3, -3) and terminal point (-1,4) and $\mathbf{p}=2\mathbf{v}-\mathbf{w}$

So $W = \langle -4, 7 \rangle$

a. Express **p** in component form

$\vec{P} = 2\langle 2,3 \rangle - \langle -4,7 \rangle$

$= \langle 4, 6 \rangle + \langle 4, -7 \rangle$

$= \langle 8, -1 \rangle$

b. Find $\|p\|$

$\|\vec{p}\| = \sqrt{8^2+1^2} = \sqrt{65}$

c. What is the unit vector form of **p**?

\# $\vec{p} = 8\vec{i} - \vec{j}$

Find the limit

* L'Hospital's

3. $\lim_{t\to 1}\left(\sqrt{t+3}i + \frac{t-1}{t^2-1}j\right)$

$= \left(\sqrt{1+3}\,\vec{i} + \frac{1}{2}\vec{j}\right)$

$= \left(2\vec{i} + \frac{1}{2}\vec{j}\right)$

multiply times conjugate (or L'Hospital's)

4. $\lim_{t\to 0}\left\langle \frac{e^t-1}{t}, \frac{\sqrt{1+t}-1}{t} \right\rangle$

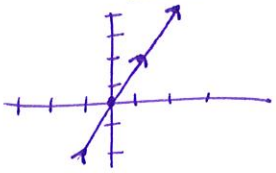
$= \langle 1, \frac{1}{2} \rangle$

Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases.

5. $r(t) = \langle t, 2t \rangle$

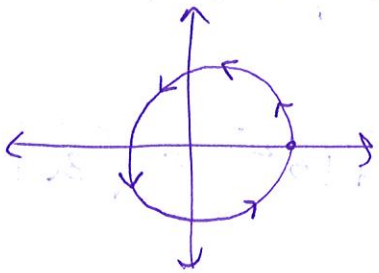
$x = t \quad y = 2t$

So $\boxed{y = 2t}$



6. $r(t) = (\sin t\ \mathrm{i} + \cos t\ \mathrm{j})$

$x = \sin t \quad y = \cos t$



circle

7. An object is moving in the xy-plane and its position after t seconds is $r(t) = (t^2 + 2t)i + (t + 4)j$

a. Find the position of the object at time t =2

$r(2) = (2^2 + 2(2))\vec{i} + (2+4)\vec{j}$

$\boxed{r(2) = 8\vec{i} + 6\vec{j}}$

b. At what time is the object at the point (15, 7)?

$x = t^2 + 2t$

$15 = t^2 + 2t$

$t = 3$

$y = t + 4$

$7 = t + 4$

$t = 3$

$\boxed{t = 3}$

c. Does the object pass through the point (20, 9)?

$x = t^2 + 2t$

$20 = t^2 + 2t$

$t \neq 5$

$y = t + 4$

$9 = t + 4$

$t = 5$

$\boxed{\text{no}}$

d. Find an equation in x and y whose graph is the path of the object.

Cartesian

$x = t^2 + 2t \qquad y = t + 4$

$t = y - 4$

$\boxed{x = (y-4)^2 + 2(y-4) \quad \text{OR} \quad x = y^2 - 6y + 8}$