CALCULUS BC
WORKSHEET 1 ON VECTORS

TEST REVIEW

Work the following on **notebook paper**. Use your calculator on problems 10 and 13c only.

1. If $x = t^2 - 1$ and $y = e^{t^3}$, find $\frac{dy}{dx}$.

2. If a particle moves in the xy-plane so that at any time $t > 0$, its position vector is $\left\langle \ln\left(t^2 + 5t\right),\ 3t^2 \right\rangle$, find its velocity vector at time $t = 2$.

3. A particle moves in the xy-plane so that at any time t, its coordinates are given by $x = t^5 - 1$ and $y = 3t^4 - 2t^3$. Find its acceleration vector at $t = 1$.

4. If a particle moves in the xy-plane so that at time t its position vector is $\left\langle \sin\left(3t - \frac{\pi}{2}\right), 3t^2 \right\rangle$, find the velocity vector at time $t = \frac{\pi}{2}$.

5. A particle moves on the curve $y = \ln x$ so that its x-component has derivative $x'(t) = t + 1$ for $t \ge 0$. At time $t = 0$, the particle is at the point (1, 0). Find the position of the particle at time $t = 1$.

6. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1 + t,\ t^3 \rangle$. If the position vector at $t = 0$ is $\langle 5,\ 0 \rangle$, find the position of the particle at $t = 2$.

7. A particle moves along the curve $xy = 10$. If $x = 2$ and $\frac{dy}{dt} = 3$, what is the value of $\frac{dx}{dt}$?

8. The position of a particle moving in the xy-plane is given by the parametric equations $x = t^3 - \frac{3}{2}t^2 - 18t + 5$ and $y = t^3 - 6t^2 + 9t + 4$. For what value(s) of t is the particle at rest?

9. A curve C is defined by the parametric equations $x = t^3$ and $y = t^2 - 5t + 2$. Write the equation of the line tangent to the graph of C at the point $(8, -4)$.

10. A particle moves in the xy-plane so that the position of the particle is given by $x(t) = 5t + 3\sin t$ and $y(t) = (8 - t)(1 - \cos t)$ Find the velocity vector at the time when the particle's horizontal position is $x = 25$.

11. The position of a particle at any time $t \ge 0$ is given by $x(t) = t^2 - 3$ and $y(t) = \frac{2}{3}t^3$.
   (a) Find the magnitude of the velocity vector at time $t = 5$.
   (b) Find the total distance traveled by the particle from $t = 0$ to $t = 5$.
   (c) Find $\frac{dy}{dx}$ as a function of x.

~~12.~~ Point $P(x, y)$ moves in the xy-plane in such a way that $\frac{dx}{dt} = \frac{1}{t+1}$ and $\frac{dy}{dt} = 2t$ for $t \ge 0$.
   SKIP
   (a) Find the coordinates of P in terms of t given that $t = 1$, $x = \ln 2$, and $y = 0$.
   (b) Write an equation expressing y in terms of x.
   (c) Find the average rate of change of y with respect to x as t varies from 0 to 4.
   (d) Find the instantaneous rate of change of y with respect to x when $t = 1$.

13. Consider the curve C given by the parametric equations $x = 2 - 3\cos t$ and $y = 3 + 2\sin t$, for $-\frac{\pi}{2} \le t \le \frac{\pi}{2}$.
   (a) Find $\frac{dy}{dx}$ as a function of t.
   (b) Find the equation of the tangent line at the point where $t = \frac{\pi}{4}$.
   (c) The curve C intersects the y-axis twice. Approximate the length of the curve between the two y-intercepts.

* also review quiz: vertical / horizontal tangents and arclength

1. $\boxed{\dfrac{dy}{dx} = \dfrac{3t^2 e^{t^3}}{2t}}$

2. $\vec{v} = \left\langle \frac{1}{t^2+5t} \cdot (2t+5) \; , \; 6t \right\rangle$

$\vec{v}(2) = \left\langle \frac{1}{14} \cdot 9 \; , \; 12 \right\rangle \quad = \boxed{\left\langle \frac{9}{14} \, , \, 12 \right\rangle}$

3. $\vec{s} = \langle t^5 - 1 \, , \; 3t^4 - 2t^3 \rangle$

$\vec{v} = \langle 5t^4 \, , \; 12t^3 - 6t^2 \rangle$

$\vec{a} = \langle 20t^3 \, , \; 36t^2 - 12t \rangle$

$\boxed{\vec{a}(1) = \langle 20 \, , \, 24 \rangle}$

4. $\vec{s} = \langle \sin(3t - \pi/2) \, , \; 3t^2 \rangle$

$\vec{v} = \langle 3\cos(3t - \pi/2) \, , \; 6t \rangle$

$\vec{v}(\pi/2) = \langle 3\cos\pi \, , \; 3\pi \rangle \quad = \boxed{\langle -3 \, , \, 3\pi \rangle}$

5. given: $\frac{dx}{dt} = t + 1$

position: $x(1) = 1 + \int_0^1 t+1 \, dt$

$= 1 + \left[\frac{1}{2}t^2 + t\right]_0^1$

$= 1 + \frac{1}{2} + 1 - 0 = 2.5$

coordinate pt of position at $t=1$: $\boxed{(2.5, \ln 2.5)}$

6. given: $\frac{dx}{dt} = 1 + t$, $\frac{dy}{dt} = t^3$

position:

$x(2) = 5 + \int_0^2 1+t \, dt$

$= 5 + \left[t + \frac{1}{2}t^2\right]_0^2$

$= 5 + 2 + 2 - 0$

$= 9$

$y(2) = 0 + \int_0^2 t^3 \, dt$

$= 0 + \left.\frac{1}{4}t^4\right]_0^2$

$= 4 - 0$

$= 4$

coordinate pt of position at $t=2$: $\boxed{(9,4)}$

7. if $xy = 10$, then $y = \frac{10}{x}$

$$\frac{d}{dt}\left(y = \frac{10}{x}\right)$$

$$\frac{dy}{dt} = \frac{-10}{x^2}\frac{dx}{dt}$$

$$3 = \frac{-10}{(2)^2}\frac{dx}{dt}$$

$$3 \cdot \frac{4}{-10} = \frac{dx}{dt}$$

$$\boxed{\frac{dx}{dt} = \frac{-12}{10} = \frac{-6}{5}}$$

8. particle at rest when velocity = 0

dx/dt + dy/dt

$$\frac{dx}{dt} = 3t^2 - 3t - 18 = 0$$

$$3(t^2 - t - 6) = 0$$

$$3(t-3)(t+2) = 0$$

$$t = -2, \underline{\underline{3}}$$

$$\frac{dy}{dt} = 3t^2 - 12t + 9 = 0$$

$$3(t^2 - 4t + 3) = 0$$

$$3(t-3)(t-1) = 0$$

$$t = 1, \underline{\underline{3}}$$

$$\boxed{t = 3}$$

9. Point : $\boxed{(8,-4)}$

Slope : $\frac{dy}{dx} = \frac{2t-5}{3t^2}$

$$\left.\frac{dy}{dx}\right|_{\underbrace{t=2}} = \frac{2(2)-5}{3(2)^2} = \boxed{\frac{-1}{12}}$$

$$8 = t^3$$

$$t = 2$$

$$-4 = t^2 - 5t + 2$$

$$t = 2$$

tangent : $\boxed{y + 4 = -\frac{1}{12}(x - 8)}$

10. $x(t) = 5t + 3\sin t$

$25 = 5t + 3\sin t$

$t = 5.446$ ← calculator

$\vec{s} = \langle 5t + 3\sin t, (8-t)(1-\cos t) \rangle$

$\vec{v} = \langle 5 + 3\cos t, (8-t)(\sin t) + (1-\cos t)(-1) \rangle$

$\boxed{\vec{v}(5.446) = \langle 7.008, -2.228 \rangle}$ ← calculator

11. $\frac{dx}{dt} = 2t$ $\frac{dy}{dt} = 2t^2$

a) $\vec{v} = \langle 2t, 2t^2 \rangle$

$\vec{v}(5) = \langle 10, 50 \rangle$

magnitude : $\sqrt{10^2 + 50^2} = \sqrt{100 + 2500} = \boxed{\sqrt{2600}}$

b) $d = \int_0^5 \sqrt{(2t)^2 + (2t^2)^2}\, dt = \int_0^5 \sqrt{4t^2 + 4t^4}\, dt = \int_0^5 \sqrt{4t^2(1+t^2)}\, dt$

$= \int_0^5 2t\sqrt{1+t^2}\, dt$ $u = 1+t^2$ $du = 2t\, dt$ $\int_1^{26} u^{1/2}\, du = \frac{2}{3} u^{3/2} \Big]_1^{26}$

$= \frac{2}{3}(26)^{3/2} - 1$

$\boxed{c}$ $x(t) = t^2 - 3$

$\downarrow$

$t^2 = x + 3$

$t = \sqrt{x+3}$

$y = \frac{2}{3}(\sqrt{x+3})^3$

$\frac{dy}{dx} = 2(\sqrt{x+3})^2 \cdot \frac{1}{2}(x+3)^{-1/2} \cdot 1$

$= \frac{2(\sqrt{x+3})^2}{2\sqrt{x+3}} = \frac{x+3}{\sqrt{x+3}} = \boxed{\sqrt{x+3}}$

12.

13.

a.

$$\boxed{\frac{dy}{dx} = \frac{2\cos t}{3\sin t} = \frac{2}{3}\cot(t)}$$

b

POINT: $\left(2-3\cos\pi/4,\ 3+2\sin\pi/4\right)$

$$\boxed{\left(2-3\sqrt{2}/2,\ 3+\cancel{2}\sqrt{2}/\cancel{2}\right)}$$

SLOPE: $\frac{dy}{dx} = \frac{2}{3}\cot(t)$

$$\left.\frac{dy}{dx}\right|_{t=} = \frac{2}{3}\cot(\pi/4) = \frac{2\cos\pi/4}{3\sin\pi/4} = \frac{2\cdot\sqrt{2}/2}{3\cdot\sqrt{2}/2} = \boxed{\frac{2}{3}}$$

tangent: $\boxed{y-(3+\sqrt{2}) = \frac{2}{3}\left(2-\frac{3\sqrt{2}}{2}\right)}$

c

curve intersects y-axis at $t =$

$$L = \int\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$$= \int\sqrt{(3\sin t)^2 + (2\cos t)^2}\,dt = \boxed{3.756}$$

to get bounds find intersection:

$0 = 2-3\cos t$

$t =$