Find the slope of the tangent line to the given polar curve at the given pt.

① $r = \cos(\theta/4)$ $\quad \theta = \pi$

$x = \cos(\theta/4)\cos\theta \quad + \quad y = \cos(\theta/4)\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos(\theta/4)\cos\theta + \sin\theta \cdot -\sin(\theta/4)\cdot 1/4}{\cos(\theta/4)\cdot -\sin\theta + \cos\theta\cdot -\sin(\theta/4)\cdot\frac{1}{4}}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{\cos(\pi/4)\cos\pi + \sin\pi\cdot -\sin(\pi/4)\cdot\frac{1}{4}}{\cos(\pi/4)\cdot -\sin\pi + \cos\pi\cdot -\sin(\pi/4)\cdot\frac{1}{4}} = \frac{\frac{\sqrt{2}}{2}\cdot -1 + \cancel{0}}{\cancel{\frac{\sqrt{2}}{2}\cdot 0} + -1\cdot -\frac{\sqrt{2}}{2}\cdot\frac{1}{4}} = \boxed{-4}$$

② $r = 4, \quad \theta = \pi/4$

$x = 4\cos\theta \quad + \quad y = 4\sin\theta$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{4\cos\theta}{-4\sin\theta} = \frac{-\cos\theta}{\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{\theta=\pi/4} = \frac{-\cos\pi/4}{\sin\pi/4} = \frac{-\sqrt{2}/2}{\sqrt{2}/2} = \boxed{-1}$$

Find the exact length of the polar curve.

③ $r = 2\cos\theta, \quad 0 \le \theta \le \pi$

$r^2 = 4\cos^2\theta$

$\frac{dr}{d\theta} = -2\sin\theta$

$\left(\frac{dr}{d\theta}\right)^2 = 4\sin^2\theta$

$$L = \int_0^{\pi}\sqrt{4\cos^2\theta + 4\sin^2\theta}\,d\theta = \int_0^{\pi}\sqrt{4(\cos^2\theta + \sin^2\theta)}\,d\theta$$

$$= \int_0^{\pi}\sqrt{4}\,d\theta = \int_0^{\pi} 2\,d\theta = 2\theta\Big]_0^{\pi}$$

$$= 2\pi - 2\cdot 0 = \boxed{2\pi}$$

④ $r = 5^{\theta}, \quad 0 \le \theta \le 2\pi$

$r^2 = 5^{2\theta}$

$\frac{dr}{d\theta} = 5^{\theta} \cdot \ln 5$

$\left(\frac{dr}{d\theta}\right)^2 = 5^{2\theta} \cdot (\ln 5)^2$

$$L = \int_0^{2\pi} \sqrt{5^{2\theta} + 5^{2\theta} \cdot (\ln 5)^2}\, d\theta = \int_0^{2\pi} \sqrt{5^{2\theta}(1+(\ln 5)^2)}\, d\theta$$

$$= \int_0^{2\pi} 5^{\theta} \underbrace{\sqrt{1+(\ln 5)^2}}_{\text{note this is just a constant!}}\, d\theta = \frac{1}{\ln 5} \cdot 5^{\theta} \cdot \sqrt{1+(\ln 5)^2}\Big]_0^{2\pi} = \boxed{\frac{1}{\ln 5} \cdot \sqrt{1+(\ln 5)^2}\left[5^{2\pi} - 1\right]}$$

⑤ $r = \theta^2, \quad [0, 2\pi]$

$r^2 = \theta^4$ $\qquad \frac{dr}{d\theta} = 2\theta$ $\qquad \left(\frac{dr}{d\theta}\right)^2 = 4\theta^2$

$$L = \int_0^{2\pi} \sqrt{\theta^4 + 4\theta^2}\, d\theta = \int_0^{2\pi} \sqrt{\theta^2(\theta^2+4)}\, d\theta = \int_0^{2\pi} \theta \underbrace{\sqrt{\theta^2+4}}_{u}\, d\theta$$

$u = \theta^2 + 4$

$\frac{du}{d\theta} = 2\theta$

$du = \quad d\theta$

$\frac{1}{2} du = \theta\, d\theta$

$$= \frac{1}{2} \int_4^{4\pi^2+4} u^{1/2}\, du = \cancel{\frac{1}{2}} \cdot \frac{\cancel{2}}{3} u^{3/2}\Big]_4^{4\pi^2+4} = \boxed{\frac{1}{3}(4\pi^2+4)^{3/2} - \frac{1}{3}(4)^{3/2}}$$

⑥ $r = 2(1+\cos\theta), \quad [0, 2\pi]$

$r^2 = 4 + 8\cos\theta + 4\cos^2\theta$

$\frac{dr}{d\theta} = -2\sin\theta$

$\left(\frac{dr}{d\theta}\right)^2 = 4 \quad {}^2\theta$

$$L = \int_0^{2\pi} \sqrt{4 + 8\cos\theta + 4\cos^2\theta + 4\sin^2\theta}\, d\theta = \int_0^{2\pi} \sqrt{4 + 8\cos\theta + 4}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{8(\cos\theta+1)}\, d\theta = \sqrt{8} \int_0^{2\pi} \sqrt{\cos\theta + 1}\, d\theta = \sqrt{8} \int_0^{2\pi} \sqrt{2 \cdot \tfrac{1}{2}(\cos\theta+1)}\, d\theta$$

* half angle formula

$$= \sqrt{8} \int_0^{2\pi} \sqrt{2\cos^2(\theta/2)}\, d\theta = \sqrt{8}\sqrt{2} \int_0^{2\pi} \cos\theta/2\, d\theta = \sqrt{16} \cdot 2\sin\theta/2 \Big]_0^{\pi}$$

$$= 8\sin\theta/2 \Big]_0^{2\pi} = 8\sin\pi - 8\sin 0 =$$