Find area of region.

1. $r = e^{-\theta/4}, \quad \pi/2 \le \theta \le \pi$

$$A = \int_{\pi/2}^{\pi} \frac{1}{2}\left(e^{-\theta/4}\right)^2 d\theta$$

$$= \frac{1}{2}\int_{\pi/2}^{\pi} e^{-\theta/2}\, d\theta$$

$$= \cancel{\frac{1}{2}} \cdot -e^{-\theta/2} \cdot \cancel{2}\Big]_{\pi/2}^{\pi}$$

$$= \left[-e^{-\pi/2}\right] - \left[-e^{-\pi/4}\right]$$

$$= \boxed{-e^{-\pi/2} + e^{-\pi/4}}$$

2. $r = \cos\theta, \quad 0 \le \theta \le \pi/6$

$$A = \int_0^{\pi/6} \frac{1}{2}\left(\cos^2\theta\right) d\theta$$

$$= \frac{1}{2}\int \frac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{4}\int 1+\cos 2\theta\, d\theta \quad = \frac{1}{4}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/6}$$

$$= \frac{1}{4}\left[\pi/6 + \frac{1}{2}\cdot\frac{1}{2}\sqrt{3}\right] = \boxed{\frac{\pi}{24} + \frac{1}{16}\sqrt{3}}$$

3. $r^2 = 9\sin 2\theta$, $r \geq 0$, $0 \leq \theta \leq \pi/2$

~~$r = \sqrt{9\sin 2\theta} = 3\sqrt{\sin 2\theta}$~~

$$\boxed{A = \int_0^{\pi/2} \frac{1}{2} \cdot 9\sin 2\theta \, d\theta}$$

$$= \frac{9}{2} \cdot -\frac{1}{2} \cdot \cos 2\theta \Big]_0^{\pi/2}$$

$$= -\frac{9}{4}(-1-1) = \boxed{9/2}$$

4. $r = \tan\theta$, $\pi/6 \leq \theta \leq \pi/3$

$$\boxed{A = \int_{\pi/6}^{\pi/3} \frac{1}{2}(\tan^2\theta) \, d\theta}$$

$$= \frac{1}{2}\int \sec^2\theta - 1 \, d\theta$$

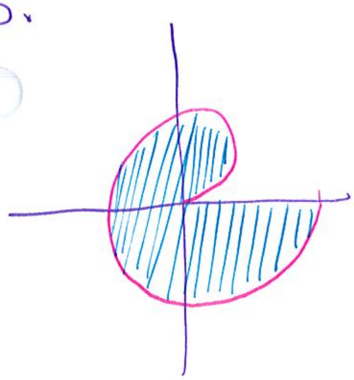
$$= \frac{1}{2}\left[\int \sec^2\theta \, d\theta - \int d\theta\right]$$

$$= \frac{1}{2}\left[\tan\theta - \theta\right] \Big]_{\pi/6}^{\pi/3}$$

$$= \frac{1}{2}\left[\frac{2}{3}\sqrt{3} - \frac{\pi}{6}\right] = \boxed{\frac{1}{3}\sqrt{3} - \frac{\pi}{12}}$$

Find the area of the shaded region:

5.



$r = \sqrt{\theta}$

$$\boxed{A = \int_0^{2\pi} \frac{1}{2} (\sqrt{\theta})^2 \, d\theta}$$

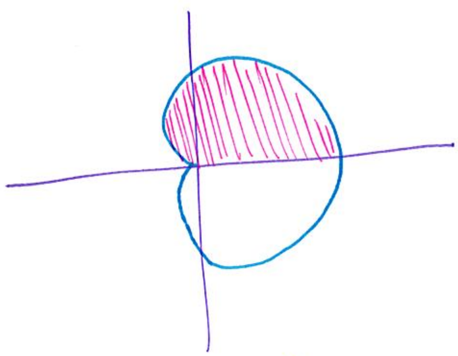
$$= \frac{1}{2} \int \theta \, d\theta$$

$$= \frac{1}{2} \cdot \frac{1}{2} \theta^2 \Big]$$

$$= \frac{\theta^2}{4} \Big]_0^{2\pi}$$

$$= \boxed{\pi^2}$$

6.



$r = 1 + \cos\theta$

$$\boxed{A = \int_0^{\pi} \frac{1}{2} (1 + \cos\theta)^2 \, d\theta}$$

$$= \frac{1}{2} \int_0^{\pi} 1 + 2\cos\theta + \cos^2\theta \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi} 1 + 2\cos\theta + \frac{1}{2}(1 + \cos 2\theta) \, d\theta$$

$$= \frac{1}{2} \int_0^{\pi} \frac{3}{2} + 2\cos\theta + \frac{1}{2}\cos 2\theta \, d\theta$$

$$= \frac{1}{2}\left[\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta\right]_0^{\pi}$$

$$= \frac{1}{2}\left(\frac{3}{2}\pi + 0 + 0\right) - \frac{1}{2}(0) = \boxed{3\pi/4}$$