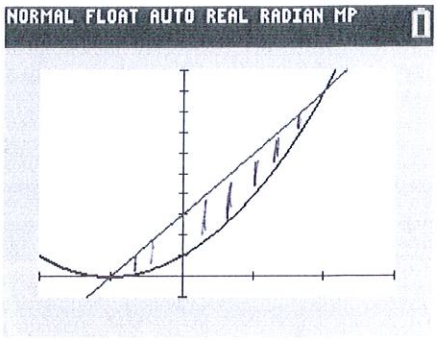
Cross Sectional Volume KEY

Find the volume of the solid enclosed between the given curves, if the cross sections perpendicular to the x-axis are...

$$y = x^2 + 2x + 1$$
$$y = 3x + 3$$



FIND BOUNDS:

$x^2 + 2x + 1 = 3x + 3$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, -1$

a. squares

$$V = \int_a^b (\text{Area})\, dx$$
$$V = \int_a^b (\text{base})^2\, dx$$

$$V = \int_{-1}^{2} [\underbrace{(3x+3) - (x^2+2x+1)}_{\text{base}}]^2\, dx$$

b. rectangles (whose height is half that of the base)

$$V = \int_a^b (\text{Area})\, dx$$
$$V = \int_a^b (\text{base})(\text{height})\, dx$$

$$V = \int_{-1}^{2} \underbrace{[(3x+3) - (x^2+2x+1)]}_{\text{base}} \cdot \underbrace{\tfrac{1}{2}[(3x+3) - (x^2+2x+1)]}_{\text{height}}\, dx$$
$$= \frac{1}{2}\int_{-1}^{2} [(3x+3) - (x^2+2x+1)]^2\, dx$$

c. circles

$$V = \int_a^b (\text{Area})\, dx$$
$$V = \int_a^b \pi r^2\, dx \qquad r = \frac{f(x)}{2}$$

$$V = \pi \int_{-1}^{2} \left[\underbrace{\frac{(3x+3) - (x^2+2x+1)}{2}}_{\text{radius}}\right]^2 dx$$

d. semi-circles

$$V = \int_a^b (\text{Area})\, dx$$
$$V = \frac{1}{2}\int_a^b \pi r^2\, dx \qquad r = \frac{f(x)}{2}$$

$$V = \frac{1}{2}\pi \int_{-1}^{2} \left[\underbrace{\frac{(3x+3) - (x^2+2x+1)}{2}}_{\text{radius}}\right]^2 dx$$