CALCULUS KEY

WORKSHEET ON VOLUME BY CROSS SECTIONS

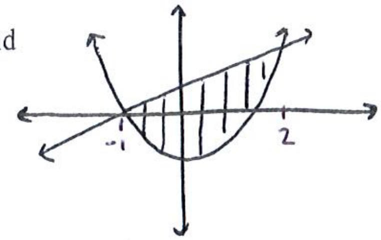
Work the following problems in space below. For each problem, find bounds, set up an integral, and then evaluate on your calculator. Give decimal answers correct to three decimal places.

1. Find the volume of the solid whose base is bounded by the graphs of $y = x + 1$ and $y = x^2 - 1$, with the indicated cross sections taken perpendicular to the x-axis.

(a) Squares

(b) Rectangles of height 1

(c) Semicircles



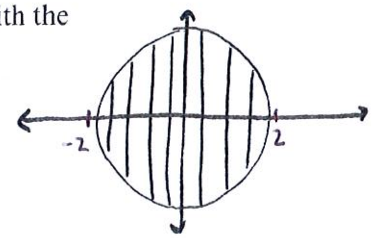
2. Find the volume of the solid whose base is bounded by the circle $x^2 + y^2 = 4$ with the indicated cross sections taken perpendicular to the x-axis.

(a) Squares

(b) Rectangles of height 3.5

(c) Semicircles

(d) Isosceles right triangles with one leg as the base of the solid

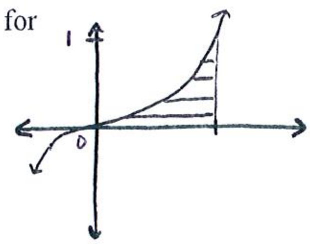


3. The base of a solid is bounded by $y = x^3$, $y = 0$, and $x = 1$. Find the volume of the solid for each of the following cross sections taken perpendicular to the y-axis.

(a) Squares

(b) Semicircles

(c) Rectangles whose height is twice the base



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1. Bounds: $x + 1 = x^2 - 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

if ⊥ to x-axis, function is in terms of x, use x bounds, use dx

a. $V = \int_{-1}^{2} \left[(x+1) - (x^2-1)\right]^2 \, dx = \boxed{8.1}$

b. $V = \int_{-1}^{2} 1 \cdot \left[(x+1) - (x^2-1)\right] \, dx = \boxed{4.5}$

c. $V = \int_{-1}^{2} \pi \cdot \frac{1}{2} \cdot \left[\frac{(x+1) - (x^2-1)}{2}\right]^2 \, dx = \boxed{1.013\pi \text{ or } 3.181}$

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2. curve: $x^2 + y^2 = 4$

$y = \pm\sqrt{4 - x^2}$

if ⊥ to x-axis, curve is in terms of x, use x-bounds, use dx

Bounds: $\pm\sqrt{4-x^2} = 0$

$x = \pm 2$

a. $V = \int_{-2}^{2} \left(2\sqrt{4-x^2}\right)^2 \, dx = \boxed{42.667}$

b. $V = \int_{-2}^{2} 3.5 \cdot \left(2\sqrt{4-x^2}\right) \, dx = \boxed{43.982}$

c. $V = \int_{-2}^{2} \pi \cdot \frac{1}{2} \left[\frac{2\sqrt{4-x^2}}{2}\right]^2 \, dx = \boxed{5.333\pi \text{ or } 16.755}$

d. $V = \int_{-2}^{2} \frac{1}{2} \cdot \left(2\sqrt{4-x^2}\right)\left(2\sqrt{4-x^2}\right) \, dx = \boxed{21.333}$

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3. if ⊥ to y-axis, curve is in terms of y, use y-bounds, use dy

curve: $y = x^3$

$x = \sqrt[3]{y}$

Bounds: $x = \sqrt[3]{y} = 1$

$y = 1$

a. $V = \int_{0}^{1} \left(1 - \sqrt[3]{y}\right)^2 \, dy = \boxed{.099 \text{ or } .1}$

b. $V = \int_{0}^{1} \pi \cdot \frac{1}{2} \cdot \left(\frac{1 - \sqrt[3]{y}}{2}\right)^2 \, dy = \boxed{.012\pi \text{ or } .039}$

c. $V = \int_{0}^{1} 2 \cdot \left(1 - \sqrt[3]{y}\right)\left(1 - \sqrt[3]{y}\right) \, dy = \boxed{.199 \text{ or } .2}$