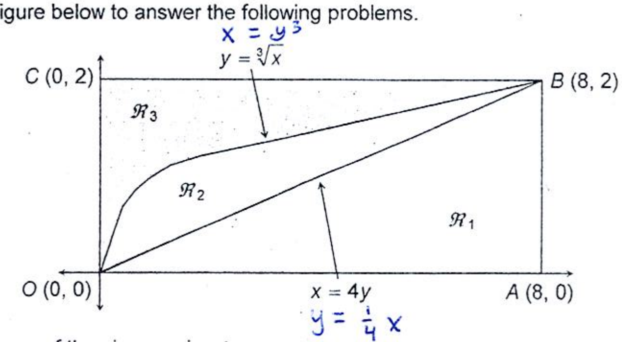
AP Calculus Name________________
Area and Volume Worksheet

Use the figure below to answer the following problems.



Find the area of the given region.

1. $\mathscr{R}_1$ 2. $\mathscr{R}_2$

3. $\mathscr{R}_3$

Find the volume generated by rotating the given region about the given line.

4. $\mathscr{R}_1$ about OA 5. $\mathscr{R}_3$ about OC

6. $\mathscr{R}_1$ about OC 7. $\mathscr{R}_2$ about OA

8. $\mathscr{R}_2$ about OC

1. $\boxed{\int_0^8 \frac{1}{4}x\,dx} = \frac{1}{8}x^2\Big]_0^8 = \frac{1}{8}(8)^2 - \frac{1}{8}(0)^2 = \boxed{8}$

2. $\boxed{\int_0^8 \left(\sqrt[3]{x} - \frac{1}{4}x\right)dx} = \frac{3}{4}x^{4/3} - \frac{1}{8}x^2\Big]_0^8 = \left(\frac{3}{4}(8)^{4/3} - \frac{1}{8}(8)^2\right) - (0) = 4$

3. $\boxed{\int_0^8 (2 - \sqrt[3]{x})\,dx} = 2x - \frac{3}{4}x^{4/3}\Big]_0^8 = \left(2(8) - \frac{3}{4}(8)^{4/3}\right) - (0) = 4$

4. $\boxed{\pi\int_0^8 \left(\frac{1}{4}x\right)^2 dx} = \pi \cdot \frac{1}{48}x^3\Big]_0^8 = \frac{\pi}{48}\cdot 8^3 - 0 = \boxed{\frac{32}{3}\pi}$

5. $\boxed{\pi\int_0^2 (y^3)^2\,dy} = \pi\frac{1}{7}y^7\Big]_0^2 = \left(\frac{\pi}{7}\cdot 2^7\right) - (0) = \boxed{\frac{128}{7}\pi}$

6. $\boxed{\pi\int_0^2 \left[(8)^2 - (4y)^2\right]dy} = \pi\left(64y - \frac{16y^3}{3}\right)\Big]_0^2$

$= \pi\left[64(2) - \frac{16(2)^3}{3} - (0)\right] = \boxed{\frac{256}{3}\pi}$

7. $\boxed{\pi\int_0^8 \left[(\sqrt[3]{x})^2 - \left(\frac{1}{4}x\right)^2\right]dx} = \pi\int_0^8 \left(x^{2/3} - \frac{1}{16}x^2\right)dx$

$= \pi\left(\frac{3}{5}x^{5/3} - \frac{1}{48}x^3\right)\Big]_0^8 = \pi\left(\frac{3}{5}(8)^{5/3} - \frac{1}{48}(8)^3 - 0\right) = \left(\frac{96}{5} - \frac{512}{48}\right)\pi$

$= \boxed{\frac{128}{15}\pi}$

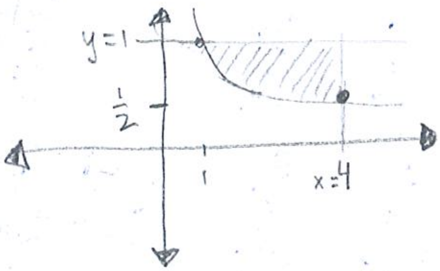
8. $\boxed{\pi\int_0^2 \left((4y)^2 - (y^3)^2\right)dy} = \pi\left(\frac{16y^3}{3} - \frac{y^7}{7}\right)\Big]_0^2$

$= \pi\left(\frac{16(2)^3}{3} - \frac{2^7}{7} - 0\right) = \pi\left(\frac{128}{3} - \frac{128}{7}\right) = \boxed{\frac{512}{21}\pi}$

$x = \frac{1}{y^2}$

Let R be the region bounded by the curves $y = \frac{1}{\sqrt{x}}$, $y = 1$, and $x = 4$.

a) Find the area of R.



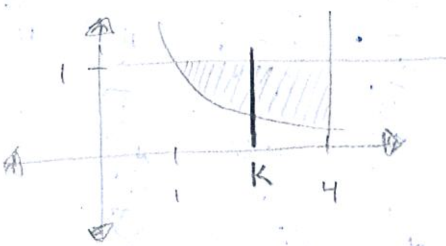
$$A = \boxed{\int_1^4 \left(1 - \frac{1}{\sqrt{x}}\right) dx} = x - 2x^{1/2}\Big]_1^4$$

$$= \left(4 - 2(4)^{1/2}\right) - (1 - 2)$$

$$= 4 - 4 + 1$$

$$= \boxed{1}$$

b) *Calculator* Suppose the line $x = k$ divides R into two regions of equal area. Find the value of k.



$$\boxed{\int_1^K \left(1 - \frac{1}{\sqrt{x}}\right) dx = \frac{1}{2}}$$

$$x - 2x^{1/2}\Big]_1^K = \frac{1}{2}$$

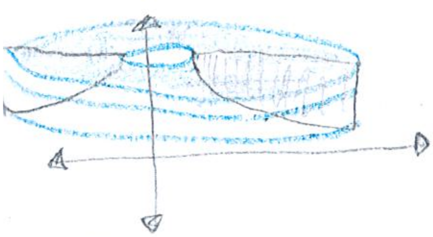
$$(K - 2K^{1/2}) - (-1) = \frac{1}{2}$$

$$K - 2K^{1/2} + \frac{1}{2} = 0 \rightarrow K - 2K^{1/2} + \frac{1}{2} = 0$$

Solve by graphing

$$\boxed{K = \sqrt{2} + \frac{3}{2}}$$

c) Find the volume of the solid generated by revolving R about the y-axis.



$$V = \boxed{\pi \int_{1/2}^{1} (4)^2 - \left(\frac{1}{y^2}\right)^2 dy}$$

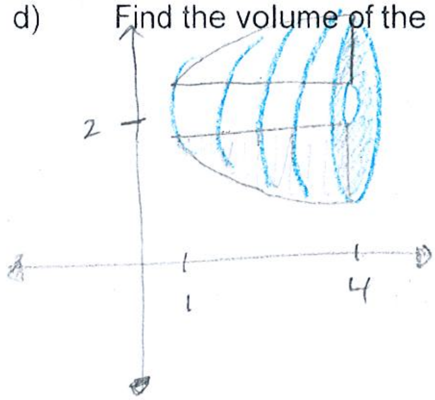
$$= \pi\left(16y + \frac{1}{3}y^{-3}\right)\Big]_{1/2}^{1}$$

$$= \pi\left[\left(16 \cdot 1 + \frac{1}{3}\right) - \left(8 + \frac{1}{3}\left(\frac{1}{2}\right)^{-3}\right)\right] = \pi\left[16 + \frac{1}{3} - 8 - \frac{8}{3}\right]$$

$$= \pi\left(8 - \frac{7}{3}\right) = \boxed{\frac{17}{3}\pi}$$

$\frac{17\pi}{3}$

d) Find the volume of the solid generated by revolving R about the line $y = 2$.



$$V = \boxed{\pi \int_1^4 \left[\left(2 - \frac{1}{\sqrt{x}}\right)^2 - (1)^2\right] dx}$$

$$= \pi \int_1^4 \left(4 - \frac{2}{\sqrt{x}} + \frac{1}{x} - 1\right) dx = \pi \int_1^4 \left(4 - 2x^{-1/2} + x^{-1} - 1\right) dx$$

$$= \pi \int_1^4 \left(3 - 2x^{-1/2} + x^{-1}\right) dx$$

$$= \pi\left(3x - 4x^{1/2} + \ln|x|\right)\Big]_1^4$$

$$= \pi\left(12 - 8 + \ln 4 - 3 + 4 - \underbrace{\cancel{\ln|1|}}_{0}\right) = \boxed{\pi\left(\ln(4) + 1\right)}$$

$\pi(2\ln 2 + 1)$