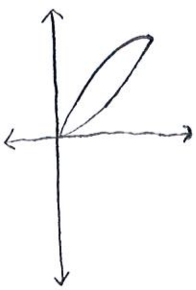
Volumes of Solids of Revolution with holes in the middle... "washer"

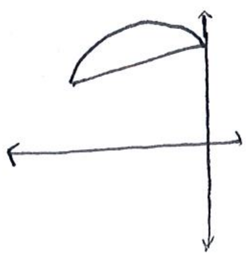
use CALCULATOR only to solve integral.

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

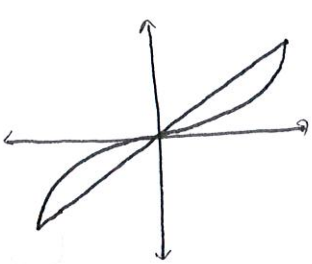
1. $y = x^2$
   $y = 4x - x^2$



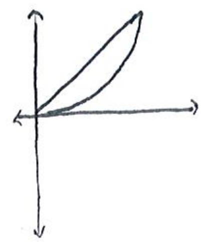
2. $y = 6 - 2x - x^2$
   $y = x + 6$



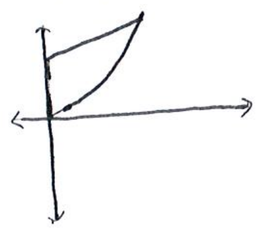
3. $y = x$
   $y = x^3$



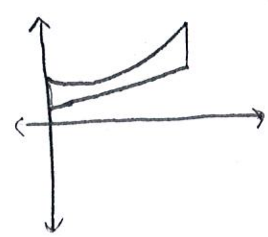
4. $y = 5x$
   $y = x^2$



5. $y = x + 6$
   $y = x^3$
   $x = 0$



6. $y = \frac{1}{2} + x^2$
   $y = x$
   $x = 0$
   $x = 2$

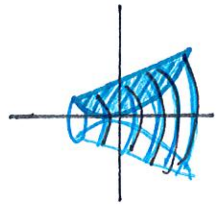


U8H3

Set up definite integral that represents each volume.

1. Rotate about x-axis

$y = x^2 + 2x + 1, \quad y = 3x + 3$



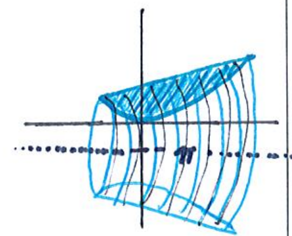
R:

r:

Volume =

2. Rotate about y = - π

$y = x^2 + 2x + 1, \quad y = 3x + 3$



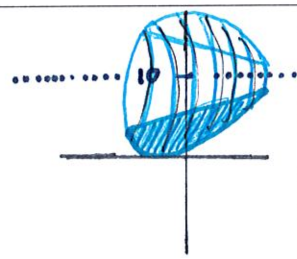
R:

r:

Volume =

3. Rotate about y = 10

$y = x^2 + 2x + 1, \quad y = 3x + 3$



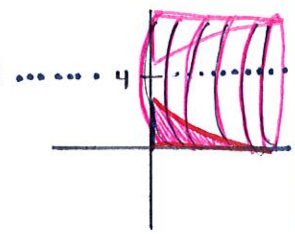
R:

r:

Volume =

4. Rotate about y = 4

$y = \dfrac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



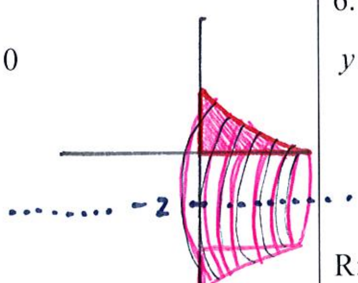
R:

r:

Volume =

5. Rotate about y = - 2

$y = \dfrac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



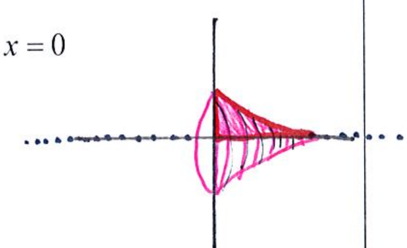
R:

r:

Volume =

6. Rotate about x- axis

$y = \dfrac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



R:

r:

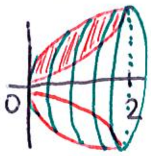
Volume =

Volumes of Solids of Revolution with holes in the middle...

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

1. $y = x^2$
$y = 4x - x^2$

$$\boxed{\begin{aligned} x^2 &= 4x - x^2 \\ 2x^2 - 4x &= 0 \\ 2x(x-2) &= 0 \\ x &= 0,\ 2 \end{aligned}} \rightarrow \text{intersections (bounds)}$$



$$\pi\int_0^2 (4x-x^2)^2 - (x^2)^2\,dx$$

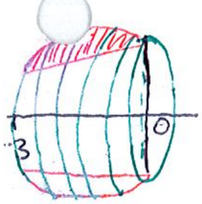
$$= \pi\int_0^2 16x^2 - 8x^3 + \cancel{x^4} - \cancel{x^4}\,dx$$

$$= \pi\left[\frac{16x^3}{3} - \frac{8x^4}{4} + \cancel{\frac{2x^5}{5}}\right]_0^2$$

$$= \pi\left(\frac{16(2)^3}{3} - 2(2)^4 + \cancel{\frac{2(2)^5}{5}}\right) - 0 = 10\frac{2}{3}\pi$$

2. $y = 6 - 2x - x^2$
$y = x + 6$

$$\boxed{\begin{aligned} 6 - 2x - x^2 &= x + 6 \\ x^2 + 3x &= 0 \\ x(x+3) &= 0 \\ x &= 0, -3 \end{aligned}} \rightarrow \text{intersections (bounds)}$$



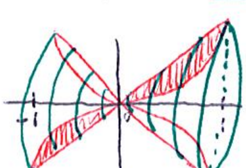
$$\int_{-3}^{0} (6-2x-x^2)^2 - (x+6)^2\,dx$$

$$= 48.6\pi$$

calculator!

3. $y = x$
$y = x^3$

$$\boxed{\begin{aligned} x &= x^3 \\ x &= 0, \pm 1 \end{aligned}} \rightarrow \text{intersections (bounds)}$$



$$2\pi\int_0^1 (x)^2 - (x^3)^2\,dx$$

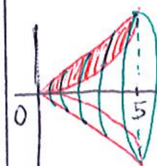
$$2\pi\int_0^1 (x^2 - x^6)\,dx$$

$$= 2\pi\left[\frac{1}{3}x^3 - \frac{1}{7}x^7\right]_0^1$$

$$= 2\pi\left[\frac{1}{3} - \frac{1}{7} - 0\right] = .381\pi$$

4. $y = 5x$
$y = x^2$

$$\boxed{\begin{aligned} 5x &= x^2 \\ x^2 - 5x &= 0 \\ x(x-5) &= 0 \\ x &= 0, 5 \end{aligned}} \rightarrow \text{intersections (bounds)}$$



$$\pi\int_0^5 (5x)^2 - (x^2 \cancel{-5x})^2\,dx$$

$$= \pi\int_0^5 \cancel{25x^2} - (x^4 - 10x^3 + \cancel{25x^2})\,dx$$

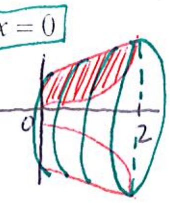
$$= \pi\int_0^5 -x^4 + 10x^3\,dx$$

$$= \pi\left[-\frac{x^5}{5} + \frac{10x^4}{4}\right]_0^5$$

$$= \pi\left(-\frac{(5)^5}{5} + \frac{10(5)^4}{4}\right) - 0 = 937.5\pi$$

5. $y = x + 6$
$y = x^3$
$\boxed{x = 0}$

$$\boxed{\begin{aligned} x + 6 &= x^3 \\ x &= 2 \end{aligned}} \rightarrow \text{intersections (bounds)}$$



$$\pi\int_0^2 (x+6)^2 - (x^3)^2\,dx$$

$$\pi\int_0^2 (x^2 - 12x + 36 - x^6)\,dx$$

$$= \pi\left[\frac{1}{3}x^3 - \frac{12x^2}{2} + 36x - \frac{1}{7}x^7\right]_0^2$$

$$= \pi\left(\frac{1}{3}(2)^3 - 6(2)^2 + 36(2) - \frac{1}{7}(2)^7\right)$$

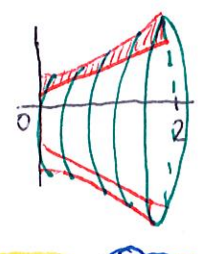
$$= 32.381\pi$$

6. $y = \frac{1}{2} + x^2$
$y = x$
$\boxed{\begin{aligned} x &= 0 \\ x &= 2 \end{aligned}}$
↓ (bounds)

$$\pi\int_0^2 \left(\frac{1}{2} + x^2\right)^2 - (x)^2\,dx$$

$$\pi\int_0^2 \left(\frac{1}{4} + \cancel{x^2} + x^4 - \cancel{x^2}\right)dx$$

$$\pi\int_0^2 \left(\frac{1}{4} + x^4\right)dx$$



$$= \pi\left[\frac{1}{4}x + \frac{1}{5}x^5\right]_0^2$$

$$= \pi\left(\frac{1}{4}(2) + \frac{1}{5}(2)^5\right) - 0 = 6.9\pi$$

$3x+3 = x^2+2x+1$

$0 = x^2 - x - 2$

$= (x-2)(x+1)$

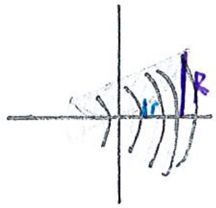
$x = 2, -1$

U8H3

Set up definite integral that represents each volume.

1. Rotate about x-axis

$y = x^2 + 2x + 1, \quad y = 3x + 3$



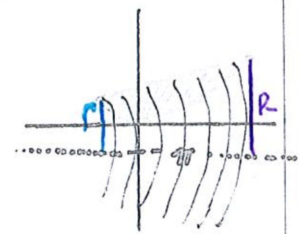
R: $3x+3$

r: x^2+2x+1

Volume = $\pi \int_{-1}^{2} (3x+3)^2 - (x^2+2x+1)^2 \, dx$

2. Rotate about y = - π

$y = x^2 + 2x + 1, \quad y = 3x + 3$



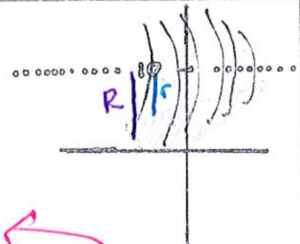
R: $3x+3+\pi$

r: $x^2+2x+1+\pi$

Volume = $\pi \int_{-1}^{2} (3x+3+\pi)^2 - (x^2+2x+1+\pi)^2 \, dx$

3. Rotate about y = 10

$y = x^2 + 2x + 1, \quad y = 3x + 3$



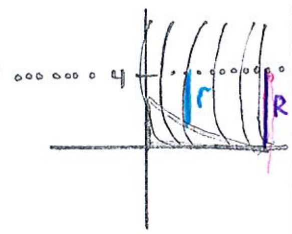
R: $10 - 3x - 3$

r: $10 - x^2 - 2x - 1$

Volume = $\pi \int_{-1}^{2} (7-3x)^2 - (9-x^2-2x)^2 \, dx$

4. Rotate about y = 4

$y = \frac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



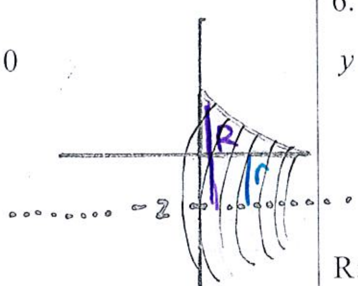
R: 4 ~~[illegible]~~

r: $4 - \frac{1}{1+x}$

Volume = $\pi \int_{0}^{3} (4)^2 - \left(4 - \frac{1}{1+x}\right)^2 dx$

5. Rotate about y = - 2

$y = \frac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



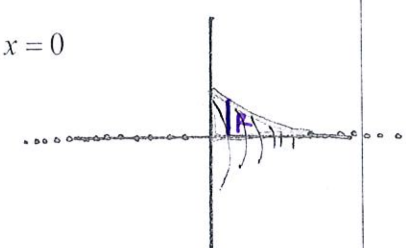
R: $\frac{1}{1+x} + 2$

r: 2

Volume = $\pi \int_{0}^{3} \left(\frac{1}{1+x} + 2\right)^2 - (2)^2 \, dx$

6. Rotate about x- axis

$y = \frac{1}{1+x}, \quad y = 0, \quad x = 3, \quad x = 0$



R: $\frac{1}{1+x} - 0$

r: 0

Volume = $\pi \int_{0}^{3} \left(\frac{1}{1+x}\right)^2 dx$