23. $\lim_{n\to\infty} 1-(0.2)^n$

$= 1-(0.2)^\infty$

$= 1-\frac{1}{5^\infty}$

$= 1-0$

$= \boxed{1 \to \text{converges}}$

24. $\lim_{n\to\infty} \frac{n^3}{n^3+1} = \frac{\infty^3}{\infty^3+1}$ ← indetermi..

$\Rightarrow \frac{3n^2}{3n^2+1}$ ← indet

$\Rightarrow \frac{6n}{6n+1}$ ← inde

$\Rightarrow \frac{6}{6} = \boxed{1 \to \text{converges}}$

25. $\lim_{n\to\infty} \frac{3+5n^2}{n+n^2} \Rightarrow$

$= \frac{10n}{1+2n} \Rightarrow \frac{10}{2}$

$= \boxed{5 \to \text{converges}}$

26. $\lim_{n\to\infty} \frac{n^3}{n+1}$

$= \frac{3n^2}{1} = 3(\infty)^2$

$= \infty \to \boxed{\text{DIVERGENT}}$

27. $\lim_{n\to\infty} e^{1/n}$

$= e^{\frac{1}{\infty}} = e^0$

$= \boxed{1 \to \text{converges}}$

28. $\lim_{n\to\infty} \frac{3^{n+2}}{5^n} =$

$= \frac{3^n \cdot 3^2}{5^n} = 9\left(\frac{3^n}{5^n}\right)$

$= 9\left(\frac{3}{5}\right)^n$

[since $-1<r<1$, $\lim_{n\to\infty} r^n = 0$]

$= 9 \cdot 0$

$= \boxed{0 \to \text{convergent}}$

29. $\lim_{n\to\infty} \tan\left(\frac{2n\pi}{1+8n}\right)$

$= \tan\left(\frac{2\pi}{8}\right)$

$= \tan \pi/4$

$= \boxed{1 \to \text{converges}}$

30. $\lim_{n\to\infty} \sqrt{\frac{n+1}{9n+1}}$

$\lim_{n\to\infty} \left[\frac{n+1}{9n+1}\right]^{1/2} = \lim_{n\to\infty}\left[\frac{1}{9}\right]^{1/2}$

$= \left(\frac{1}{9}\right)^{1/2} = \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}}$

$= \boxed{\frac{1}{3} \to \text{converges}}$

31. $\lim_{n\to\infty} \frac{n^2}{\sqrt{n^3+4n}}$

ugh... l'Hospital's is too much work here...

~~$= \frac{2n}{\frac{1}{2}(n^3+4n)^{-1/2}\cdot(3n^2+4)}$~~

simplify

~~$= \frac{2n}{\frac{3n^2+4}{2\sqrt{n^3+4n}}}$~~ simplify ~~$= \frac{4n\sqrt{n^3+4n}}{3n^2+4}$~~

~~$=$~~

take two:

$\frac{\lim_{n\to\infty} n^2}{\lim_{n\to\infty}\sqrt{n^3+4n}} = \frac{\lim_{n\to\infty} n}{\lim_{n\to\infty}\frac{1}{2}(n^3+4n)^{-1/2}\cdot(3n^2+4)}$

$= \frac{\lim_{n\to\infty} n \cdot \boxed{\lim_{n\to\infty} 2\sqrt{n^3+4n}} \to \infty}{\lim_{n\to\infty} 3n^2+4}$

$= \frac{1}{6n+4} \cdot \infty = \infty \to \boxed{\text{DIVERGES}}$

32. $\lim_{n\to\infty} e^{\frac{2n}{n+2}}$

$= \lim_{n\to\infty} e^{\frac{2}{1}}$

$= \boxed{e^2 \to \text{converges}}$

33. $\lim_{n\to\infty} \frac{(-1)^n}{2\sqrt{n}} = \frac{1}{2}\lim_{n\to\infty}\frac{(-1)^n}{n^{1/2}}$

$= \frac{1}{2}\cdot\frac{\pm 1}{\infty} = \frac{1}{2}\cdot 0$

$= \boxed{0 \to \text{converges}}$

34. $\lim_{n\to\infty} \frac{(-1)^{n+1} n}{n+\sqrt{n}}$

note: as $\lim_{n\to\infty} a_n = 1$ for odd terms

& as $\lim_{n\to\infty} a_n = -1$ for even terms

oscilating...

$\boxed{\text{DIVERGES}}$

35. $a_n = \cos(n/2)$

$\lim_{n\to\infty} \cos(\infty/2) = \cos(\infty) =$ oscilating between -1 & $+1$

$\boxed{\text{DIVERGES}}$

36. $a_n = \cos(2/n)$

$\lim_{n\to\infty} \cos(2/\infty) = \cos(0) = \boxed{1 \to \text{CONVERGES}}$

37. $a_n = \left\{\frac{(2n-1)!}{(2n+1)!}\right\}$

$\lim_{n\to\infty} \frac{\cancel{(2n-1)!}}{(2n+1)(2n)\cancel{(2n-1)!}} = \frac{1}{(2n+1)(2n)} = \frac{1}{\infty} = \boxed{0 \to \text{CONVERGES}}$

38. $a_n = \left\{\frac{\ln(n)}{\ln(2n)}\right\}$

$\lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{2n}\cdot 2} = \frac{\frac{1}{n}}{\frac{1}{n}} = \boxed{1 \to \text{CONVERGES}}$

39. $a_n = \left\{\frac{e^n + e^{-n}}{e^{2n}-1}\right\}$

$\lim_{n\to\infty} \frac{e^n + e^{-n}}{e^{2n}-1} = \frac{\frac{e^n}{e^n}\cdot e^n + \frac{1}{e^n}}{e^{2n}-1} = \frac{\frac{e^{2n}+1}{e^n}}{e^{2n-1}} = \frac{e^{2n+1}}{e^n \cdot e^{2n-1}}$

$= \frac{\cancel{e^{2n}}\cdot e^1}{e^n \cdot \cancel{e^{2n}} \cdot e^{-1}} = \frac{e}{e^n/e} = \frac{e^2}{e^n} = \frac{e^2}{e^\infty} = \boxed{0 \to \text{converges}}$

40. $a_n = \dfrac{\tan^{-1} n}{n}$

$$\lim_{n\to\infty} \frac{\tan^{-1} n}{n} = \frac{\lim_{n\to\infty} \tan^{-1} n}{\lim_{n\to\infty} n} = \frac{\pi/2}{\infty} = \boxed{0 \rightarrow \text{CONVERGES}}$$

41. $a_n = n^2 e^{-n}$

$$\lim_{n\to\infty} \frac{n^2}{e^n} \Rightarrow \frac{2n}{e^n} \rightarrow \frac{2}{e^n} = \frac{2}{e^\infty} = \boxed{0 \rightarrow \text{CONVERGES}}$$

42. $a_n = \ln(n+1) - \ln(n)$

$$\lim_{n\to\infty} \cancel{\frac{\ln(n+1)}{\ln(n)} = \frac{\frac{1}{n+1}}{\frac{1}{n}} = \frac{n}{n+1}} \ \ln\left(\frac{n+1}{n}\right) = \ln\left(1+\frac{1}{n}\right)$$

$$= \ln\left(1+\frac{1}{\infty}\right) = \ln(1+0)$$

$$= \ln(1) = \boxed{0 \rightarrow \text{CONVERGES}}$$

43. $a_n = \dfrac{\cos^2 n}{2^n}$

$$\lim_{n\to\infty} \frac{\cos^2 n}{2^n} = \frac{\text{constant}}{\infty} = \boxed{0 \rightarrow \text{CONVERGES}}$$

THINK... oscilating btwn 2 constants

THINK.... going to infinity

44. $a_n = \sqrt[n]{2^{1+3n}}$

$$\lim_{n\to\infty} (2^{1+3n})^{1/n} = (2^1 \cdot 2^{3n})^{1/n} = 2^{1/n} \cdot 2^{3n/n}$$

$$= 2^{1/n} \cdot 2^3 = 8 \cdot 2^{1/\infty} = 8 \cdot 2^0 = 8 \cdot 1 = \boxed{8 \rightarrow \text{converges}}$$