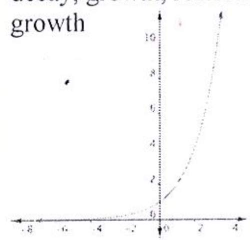
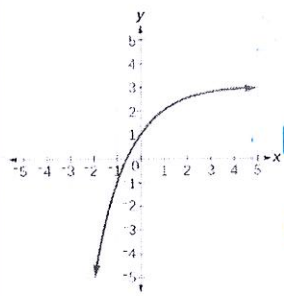
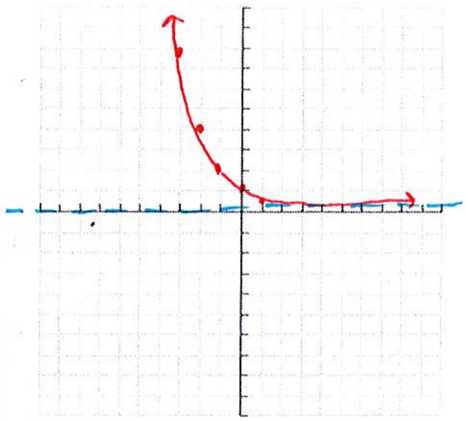
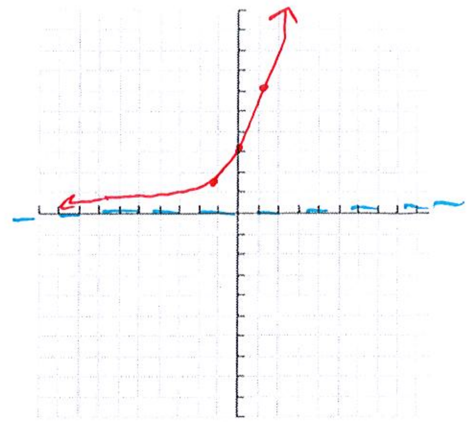
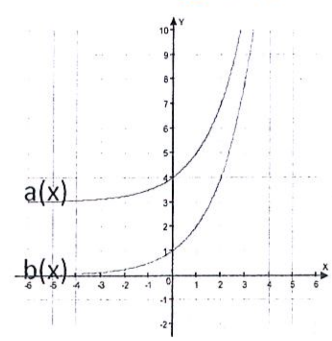
Algebra 1
Unit 4 Review Guide

Name: Key
Date:

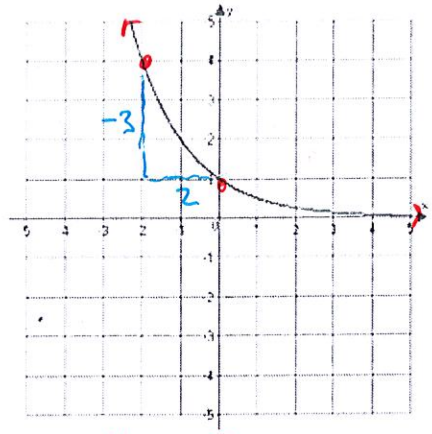
Exponential Functions Unit Review

Skill	Things to remember	Examples	
1. Determine if representations are exponential. Explain why or why not	Exponential Functions: -Variable in exponent -Constant Ratios -Graph is a curve Linear Functions: -Constant differences -Graph is a line	a. Tell if the following are exponential decay, growth, refelcted decay, or reflected growth  growth  reflected decay	b. Determine if the equations are linear or exponential: a. $y = 3^x - 4$ exponential b. $y = 2x - 3$ linear c. $y = 6^{2x}$ exponential
2. Determine if a function is exponential growth or decay and explain why.	$0 < b < 1$: Decay $b > 1$: Growth	a. $y = \underbrace{.75}_{a}\underbrace{\left(\frac{3}{2}\right)}_{b}^{x}$ Growth since $\frac{3}{2} > 1$	b. $y = \underbrace{\left(\frac{1}{2}\right)}_{b}^{x}$ $a=1$ decays since $\frac{1}{2}$ is between 0 & 1
		c. $Y = \underbrace{3}_{a}(\underbrace{2}_{b})^x$ Growth since $2 > 1$	d. $Y = \underbrace{3}_{a}(\underbrace{1-.5}_{b})^x$ $b = 0.5$ decays since 0.5 is between 0 + 1

3. Graph an exponential function.	$y = ab^x$ Create a table with values and graph. Remember to represent the asymptote as a dotted line.	a. Graph: $f(x)=\left(\frac{1}{2}\right)^x$  x | y -3 | 8 -2 | 4 -1 | 2 0 | 1 1 | 1/2	b. Graph: $f(x)=3\bullet 2^x$  x | y -1 | 1.5 0 | 3 1 | 6 2 | 12
4. Describe the transformations of an exponential function.	$f(x) = a(b)^{x-h} + k$ **a** stretches or shrinks AND reflects **k** moves the function up (+) and down (-) **h** moves the function left (+) and right (-) The new asymptote is the line y = k.	a. Given the function $f(x) = 2^x$ write a new equation after a transformation of left 7 and up 3. $f(x) = 2^{x+7} + 3$ c. Describe the transformation $h(x) = 10^x$ to $k(x) = 4(10)^{x+1} - 5$. - vertical stretch of 4 - left 1 - down 5	b. Given the function $g(x) = 2^x$, write a new equation after a transformation of right 9 and reflect across the x-axis. $g(x) = -2^{x-9}$ d. Describe the transformation from a(x) to b(x). down 3 

5. Determine characteristics of exponential functions.

a.



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

x-Intercept: none

y-intercept: $(0, 1)$

Interval of Increase: none

Interval of Decrease: $(-\infty, \infty)$

Asymptote: $y = 0$

End Behavior:

as $x \to -\infty$, $f(x) \to$ ∞

as $x \to \infty$, $f(x) \to$ 0

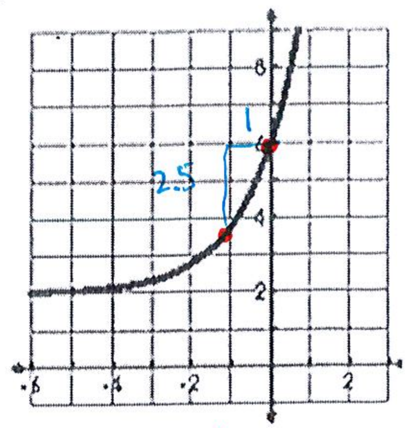
ROC over interval -2 to 0:

$x_1 = -2$ $\quad y_1 = 4$

$x_2 = 0$ $\quad y_2 = 1$

$$\frac{1-4}{0--2} = \boxed{\frac{-3}{2}} = \frac{\text{rise}}{\text{run}}$$

b.



Domain: $(-\infty, \infty)$

Range: $[2, \infty)$

x-Intercept: none

y-intercept: $(0, 6)$

Interval of Increase: $(-\infty, \infty)$

Interval of Decrease: none

Asymptote: $y = 2$

End Behavior:

as $x \to -\infty$, $f(x) \to$ 2

as $x \to \infty$, $f(x) \to$ ∞

ROC over interval -1 to 0:

$x_1 = -1$ $\quad y_1 = 3.5$

$x_2 = 0$ $\quad y_2 = 6$

$$\frac{6-3.5}{0--1} = \frac{2.5}{1} = \frac{\text{rise}}{\text{run}}$$

$$= \boxed{2.5} =$$

6. Determine the y-intercept and asymptote from an equation	You can always substitute 0 in for x to find a y-intercept Asymptote: y = k No 'k' value, the asymptote is y = 0.	a. Determine the y-intercept and asymptote of the function y = $3(2)^x$. y-int: $y = 3(2)^0 = 3$ $\boxed{(0,3)}$ asymp.: $\boxed{y = 0}$	b. Determine the y-intercept and asymptote of the function $y = 4(\frac{1}{2})^x - 2$. y-int: $y = 4(\frac{1}{2})^0 - 2 = 2$ $\boxed{(0,2)}$ asymp.: $\boxed{y = -2}$
7. Determine the growth/decay factor and percent.	(1 + r) and (1 – r) represent the growth and decay factors	a. $y = 3(1.25)^x$ Determine if the function is growth or decay: $\boxed{\text{Growth}}$ Factor: $\boxed{1.25}$ Rate: $1 + r = 1.25$ $\boxed{r = .25 = 25\%}$	b. $y = 2(.84)^x$ Determine if the function is growth or decay: $\boxed{\text{decay}}$ Factor: $\boxed{.84}$ Rate: $1 - r = .84$ $\boxed{r = .16 = 16\%}$

8. Applications of exponential functions.	$y = p(1+r)^t$ $y = p(1-r)^t$ $A = P\left(1+\frac{r}{n}\right)^{nt}$	a. Luke Duke deposits \$2000 into a bank account that pays 5% interest compounded monthly. Find the balance in the account after 4 years. Equation: $y = 2000\left(1+\frac{.05}{12}\right)^{12\cdot 4}$ Solution: \$2441.79	b. The value of the Barbie Dream House is \$125,000. This house is a prime location and appreciates (increases in value at a rate of 7% per year. How much will the Barbie Dream House be worth in 5 years? Equation: $125{,}000(1+.07)^5$ Solution: \$175,318.97
		c. A certain radioactive element decays at a rate of 21% per month. If the starting amount was 32 ounces, how much will be left after **1 year**? = 12 months Equation: $y = 32(1-.21)^{12}$ Solution: 1.89	d. Michael is offered two jobs – Job A, which offers him a starting salary of \$20,000 a year with a 5% raise each year. ~~[illegible]~~ Job B, which offers him a starting salary of \$25,000, ~~[illegible]~~ and a 3% raise each year. Michael plans to work to work at the job for 7 years. Which job should he pick and why? Job A: $20{,}000(1+.05)^7 =$ \$28,142.01 Job B: $25{,}000(1+.03)^7 =$ \$30,746.87 Job B is better in long run!
9. Solving Exponential Functions	• Must have SAME base • Set exponents = (don't forget to distribute) • Solve for x	• $5^{3x+1} = 5^{x-9}$ $3x+1 = x-9$ $2x = -10$ $x = -5$	• $3^{x-8} = 9^x$ $\cancel{3}^{x-8} = \cancel{3}^{2x}$ $x-8 = 2x$ $-8 = x$ $x = -8$
		• $4^{3x} = 8^{x+1}$ $\cancel{(2)}^{2\cdot 3x} = \cancel{2}^{3\cdot(x+1)}$ $6x = 3x+3$ $3x = 3$ $x = 1$	$4^{4x+8} = \left(\frac{1}{4}\right)^{x-18}$ $\cancel{4}^{x+8} = \cancel{4}^{-1(x-18)}$ $x+8 = -x+18$ $2x = 10$ $x = 5$

10. Geometric Sequences

Geometric Explicit Formula: $a_n = a_1 (r)^{n-1}$

Tell if the following is Geometric or Arithmetic

a. 8, 5, 2, -1… $\boxed{\text{Arithmetic}}$

$-3 \quad -3 \quad -3$

b. 2, 6, 18, 54… $\boxed{\text{Geometric}}$

$\times 3 \quad \times 3 \quad \times 3$

Create an Explicit formula and then use it to find a certain term.

c. -81, 27, -9, 3, -1

Explicit formula: $\boxed{a_n = -81\left(-\frac{1}{3}\right)^{n-1}}$

$a_8 =$ $a_8 = -81\left(-\frac{1}{3}\right)^{8-1} = \boxed{1/27 = .037}$

d. 4, 12, 36, 108,…

Explicit formula: $\boxed{a_n = 4(3)^{n-1}}$

$a_9 =$

$a_9 = 4(3)^{9-1} = \boxed{26{,}244}$

Joe sells coffee at his work place and has recorded his weekly sales below.

week	Sales
1	50. 30
2	62.10
3	76.67

$r = \frac{a_2}{a_1} = 1.23$

Explicit formula: $\boxed{a_n = 50.30(1.23)^{n-1}}$

If the same trend continues, how much will ~~he~~ make in week 7? $a_n = 50.30(1.23)^{7-1} = \boxed{\$174.18}$