17–26 Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

17. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$

18. $4 + 3 + \frac{9}{4} + \frac{27}{16} + \cdots$

19. $10 - 2 + 0.4 - 0.08 + \cdots$

20. $2 + 0.5 + 0.125 + 0.03125 + \cdots$

21. $\sum_{n=1}^{\infty} 6(0.9)^{n-1}$

22. $\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}}$

23. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

24. $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

25. $\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

26. $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$

17. $\sum_{n=1}^{\infty} 3\left(\overbrace{-\frac{4}{3}}^{r}\right)^{n-1}$ diverges since $|r| \geq 1$

18. $\sum_{n=1}^{\infty} 4\left(\overbrace{\frac{3}{4}}^{r}\right)^{n-1}$ converges since $|r| < 1$

sum $= \frac{4}{1-3/4} = \frac{4}{1/4} = 16$

19. $\sum_{n=1}^{\infty} 10\left(\overbrace{-\frac{1}{5}}^{r}\right)^{n-1}$ converges since $|r| < 1$

Sum $= \frac{10}{1+\frac{1}{5}} = \frac{10}{6/5} = \frac{50}{6} = \frac{25}{3}$

20. $\sum_{n=1}^{\infty} 2\left(\overbrace{\frac{1}{4}}^{r}\right)^{n-1}$ converges since $|r| < 1$

sum $= \frac{2}{1-1/4} = \frac{2}{3/4} = \frac{8}{3}$

21. converges since $|r| < 1$

sum $= \frac{6}{1-0.9} = \frac{6}{1/10} = 60$

22. $\sum_{n=1}^{\infty} \frac{10^n}{(-9)^{n-1}} = \sum \frac{10(10^{n-1})}{(-9)^{n-1}} = 10\sum\left(\overbrace{-\frac{10}{9}}^{r}\right)^{n-1}$

diverges since $|r| > 1$

23. $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n} = \sum \frac{(-3)^{n-1}}{4 \cdot 4^{n-1}} = \frac{1}{4} \sum \left(\overbrace{-\frac{3}{4}}^{r}\right)^{n-1}$ converges since $|r| < 1$

sum $= \frac{1}{4} \cdot \frac{1}{1+3/4} = \frac{1}{4} \cdot \frac{1}{7/4} = \frac{4}{4 \cdot 7} = \frac{1}{7}$

---

24. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n} = \sum \left(\overbrace{\frac{1}{\sqrt{2}}}^{r}\right)^n$ converges since $|r| < 1$

sum $= \frac{1}{1 - \frac{1}{\sqrt{2}}} = \frac{\sqrt{2}}{\sqrt{2} - 1} = 2 + \sqrt{2}$

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25. $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}} = \sum \frac{\pi^n}{3^n \cdot 3} = \frac{1}{3} \sum \frac{\pi^n}{3^n} = \frac{1}{3} \sum \left(\overbrace{\frac{\pi}{3}}^{r}\right)^n$

diverges since $|r| \geq 1$

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26. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}} = \sum \frac{e^n}{3^n \cdot 3^{-1}} = 3 \sum \frac{e^n}{3^n} = 3 \sum \left(\overbrace{\frac{e}{3}}^{r}\right)^n$

converges since $\left|\frac{e}{3}\right| < 1$

sum $= 3 \cdot \frac{e}{1 - e/3} = \frac{3e}{3 - e/3} = \frac{9e}{3 - e}$

27–42 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

27. $\frac{1}{3}+\frac{1}{6}+\frac{1}{9}+\frac{1}{12}+\frac{1}{15}+\cdots$

28. $\frac{1}{3}+\frac{2}{9}+\frac{1}{27}+\frac{2}{81}+\frac{1}{243}+\frac{2}{729}+\cdots$

29. $\sum_{n=1}^{\infty}\frac{n-1}{3n-1}$

30. $\sum_{k=1}^{\infty}\frac{k(k+2)}{(k+3)^2}$

31. $\sum_{n=1}^{\infty}\frac{1+2^n}{3^n}$

32. $\sum_{n=1}^{\infty}\frac{1+3^n}{2^n}$

33. $\sum_{n=1}^{\infty}\sqrt[n]{2}$

34. $\sum_{n=1}^{\infty}[(0.8)^{n-1}-(0.3)^n]$

35. $\sum_{n=1}^{\infty}\ln\left(\frac{n^2+1}{2n^2+1}\right)$

36. $\sum_{n=1}^{\infty}\frac{1}{1+(\frac{2}{3})^n}$

27. $\sum_{n=1}^{\infty}\frac{1}{3n} = \frac{1}{3}\sum_{n=1}^{\infty}\frac{1}{n}$

this is constant multiple of harmonic series... so it diverges

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28. notice: $\left(\frac{1}{3}+\frac{1}{27}+\frac{1}{243}+\cdots\right)+\left(\frac{2}{9},\frac{2}{81},\frac{2}{729}\cdots\right)$

$\left[\sum \frac{1}{3}\left(\frac{1}{9}\right)^{n-1}\right. + \left[\sum \frac{2}{9}\left(\frac{1}{9}\right)^{n-1}\right.$

series converges since $|r|<1$

sum: $\frac{\frac{1}{3}}{1-\frac{1}{9}} = \frac{3}{8} + \frac{2/9}{1-\frac{1}{9}} = \frac{1}{4}$

$\frac{1}{4}+\frac{3}{8}=\boxed{5/8}$

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29. diverges by test for Divergence since $\lim_{n\to\infty}\frac{n-1}{3n-1}=\frac{1}{3}\neq 0$

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30. diverges by test for Divergence since

$\lim_{n\to\infty}\frac{k(k+2)}{(k+3)^2}=1\neq 0$

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31. $\sum\frac{1}{3^n}+\sum\frac{2^n}{3^n} = \sum\frac{1}{3^n}+\sum\left(\frac{2}{3}\right)^n = \sum\left(\frac{1}{3}\right)^n+\sum\left(\frac{2}{3}\right)^n$

since both $|r|<1$, series converges

sum: $\frac{\frac{1}{3}}{1-\frac{1}{3}} + \frac{2/3}{1-2/3}$

$\frac{1}{2} + 2 = \boxed{\frac{5}{2}}$

---

32. $\sum\frac{1}{2^n}+\frac{3^n}{2^n} = \sum\left(\frac{1}{2}\right)^n+\sum\left(\frac{3}{2}\right)^n$

good $|r|<1$ bad $|r|\geq 1$

DIVERGES

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33. $\lim_{n\to\infty}a_n = \lim_{n\to\infty}\sqrt[n]{2} = \lim_{n\to\infty}2^{1/n} = 2^{1/\infty} = 2^0 =$

DIVERGES $=1\neq 0$

34. $\sum \left(\frac{4}{5}\right)^{n-1} - \sum \left(\frac{3}{10}\right)^{n}$

$|r| < 1$ $\qquad$ $|r| < 1$

So series converges.

sum: $\frac{1}{1-4/5} - \frac{3/10}{1-3/10} = 5 - \frac{3}{7} = \boxed{\frac{32}{7}}$

35. $\lim_{n\to\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right) = \lim_{n\to\infty} \ln\left(\frac{2n}{4n}\right) = \ln(2/4)$

$= \ln(1/2) \neq 0$

DIVERGES

36. $\lim_{n\to\infty} \frac{1}{1+(2/3)^n} = \frac{1}{1+(2/3)^\infty} = \frac{1}{1+0} = 1 \neq 0$

DIVERGES