21–26 Determine whether the series is convergent or divergent.

21. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ 22. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

23. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ 24. $\sum_{n=3}^{\infty} \frac{n^2}{e^n}$

25. $\cancel{\sum_{n=1}^{\infty} \frac{1}{n^2+n^3}}$ omit #25 26. $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$

21. $\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ } The function $y = \frac{1}{x\ln x}$ is continuous, positive, and decreasing so Integral Test can be used

$\int_2^{\infty} \frac{1}{x\ln x}\,dx$ $\left[\begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array}\right]$

$\lim_{t\to\infty} \int_{\ln 2}^{\ln t} \frac{1}{u}\,du = \ln|u| \Big]_{\ln 2}^{\ln t}$

$= \ln|t| - \ln\ln 2$

$= \ln\infty - \ln\ln 2$

$=$ $\boxed{\text{DIVERGES}}$

22. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ } The function $y = \frac{1}{x(\ln x)^2}$ is continuous, positive, and decreasing so Integral Test applies

$\int_2^{\infty} \frac{1}{x(\ln x)^2}\,dx$ $\left[\begin{array}{l} u = \ln x \\ du = \frac{1}{x}dx \end{array}\right]$

$\lim_{t\to\infty} \int_{\ln 2}^{\ln t} u^{-2}\,du = -u^{-1}\Big]_{\ln 2}^{\ln t} = -(\ln t)^{-1} + (\ln 2)^{-1}$

$= \frac{-1}{\ln t} + \frac{1}{\ln 2} = \frac{-1}{\infty} + \frac{1}{\ln 2}$

$=$ $\boxed{\text{converges } \frac{1}{\ln 2}}$

23. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ } The function $y = \frac{e^{1/x}}{x^2}$ is continuous, positive, and decreasing so Integral Test applies

$\int_1^{\infty} \frac{e^{1/x}}{x^2}\,dx = \lim_{t\to\infty} \int_1^t \frac{e^{1/x}}{x^2}\,dx$ $\left[\begin{array}{l} u = 1/x \\ du = -\frac{1}{x^2}dx \\ -du = \frac{1}{x^2}dx \end{array}\right]$ $= \int_1^{1/t} -e^u\,du$

$= -e^u\Big]_1^{1/t} = -e^{1/t} + e^1 = e - e^{1/\infty} = e - e^0 =$ $\boxed{e - 1 \rightarrow \text{converges}}$

24. $\sum_{n=3}^{\infty} \frac{n^2}{e^n}$ } The function $y = \frac{x^2}{e^x}$ is contin. positive, + decreasing so Int. Test applies

$\int_1^{\infty} \frac{x^2}{e^x}\,dx = \lim_{t\to\infty} \int_1^t \frac{x^2}{e^x}\,dx$ let: $u = x^2$, $du = 2x\,dx$; $v = -e^{-x}$, $dv = e^{-x}\,dx$

$= x^2 \cdot -e^{-x} - \int^t -e^{-x} \cdot 2x\,dx = -x^2e^{-x} + \int_1^t 2xe^{-x}\,dx$ let $u = 2x$, $du = 2\,dx$; $v = -e^{-x}$, $dv = e^{-x}dx$

$= -x^2e^{-x} - 2xe^{-x} - \int_1^t -2e^{-x}\,dx$

$= -x^2e^{-x} - 2xe^{-x} - 2e^{-x}\Big]_1^t = (-t^2e^{-t} - 2te^{-t} - 2e^{-t}) - (-(1)^2e^{-1} - 2(1)e^{-1} - 2e^{-1})$

$\boxed{\text{converges}}$

26. $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$ } Function $y = \frac{x}{x^4+1}$ is cont, positive, decr. so Integral Test applies

$$\int_1^{\infty} \frac{x}{x^4+1}\,dx = \lim_{t\to\infty} \int_1^t \frac{x}{x^4+1}\,dx = \int_1^t \frac{x}{1+(x^2)^2}\,dx$$

$$= \arctan(x^2) \cdot \frac{1}{2} \Big]_1^t = \frac{1}{2}\arctan(t)^2 - \frac{1}{2}\arctan(1)^2$$

$$= \frac{1}{2}\arctan\infty - \frac{1}{2}\cdot\frac{\pi}{4}$$

$$= \frac{1}{2}\cdot\frac{\pi}{2} - \frac{\pi}{8} = \frac{\pi}{4} - \frac{\pi}{8} = \boxed{\frac{\pi}{8} \text{ converges}}$$

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25. $\sum_{n=1}^{\infty} \frac{1}{x^2+x^3}$ } Function $y = \frac{1}{x^2+x^3}$ is continuous, positive, and decreasing so Integral Test applies

$$\int_1^{\infty}$$