1. a. since $a_n > b_n$ with $\sum b_n$ diverging, we can't say anything about the convergence of $\sum a_n$

b. we can say $\sum a_n$ converges since $a_n < b_n$

2. a. we can say $\sum a_n$ diverges since $a_n > b_n$

b. we can't say anything about the divergence of $\sum a_n$ since $a_n < b_n$

3. compare $\frac{n}{2n^3+1}$ and $\frac{n}{2n^3}$

since $\frac{n}{2n^3+1} < \frac{n}{2n^3}$ AND we know that

$\sum_{n=1}^{\infty} \frac{n}{2n^3} = \frac{1}{2}\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges ~~since~~ by p-series test with $p = 2 > 1$,

then $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$ also converges by comparison test

4. compare $\frac{n^3}{n^4-1}$ and $\frac{n^3}{n^4}$

since $\frac{n^3}{n^4-1} > \frac{n^3}{n^4}$ AND we know that

$\sum_{n=1}^{\infty} \frac{n^3}{n^4} = \sum_{n=1}^{\infty} \frac{1}{n}$ diverges by p-series test with $p = 1 \leq 1$, [OR by harmonic series]

then $\sum \frac{n^3}{n^4-1}$ also diverges by comparison test

5. compare $\frac{n+1}{n\sqrt{n}}$ and $\frac{n}{n\sqrt{n}}$

since $\frac{n+1}{n\sqrt{n}} > \frac{n}{n\sqrt{n}}$ AND we know that

$\sum_{n=1}^{\infty} \frac{n}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{n}{n^{3/2}} = \sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ diverges by p-series test with $p = 1/2 \le 1$,

then $\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$ also diverges by comparison test

6. compare $\frac{n-1}{n^2\sqrt{n}}$ and $\frac{n}{n^2\sqrt{n}}$

since $\frac{n-1}{n^2\sqrt{n}} < \frac{n}{n^2\sqrt{n}}$ AND we know that

$\sum_{n=1}^{\infty} \frac{n}{n^2\sqrt{n}} = \sum_{n=1}^{\infty} \frac{n}{n^2 \cdot n^{1/2}} = \sum_{n=1}^{\infty} \frac{n}{n^{5/2}} = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by p-series test w/ $p = \frac{3}{2} > 1$,

then $\sum_{n=1}^{\infty} \frac{n-1}{n\sqrt{n}}$ also converges by comparison test

7. compare $\frac{9^n}{3+10^n}$ and $\frac{9^n}{10^n}$

since $\frac{9^n}{3+10^n} < \frac{9^n}{10^n}$ AND we know that

$\sum_{n=1}^{\infty} \frac{9^n}{10^n} = \sum_{n=1}^{\infty} \left(\frac{9}{10}\right)^n$ converges since $r = \frac{9}{10}$ and $|r| < 1$,

then $\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ also converges by comparison test

8. compare $\frac{6^n}{5^{n-1}}$ and $\frac{6^n}{5^n}$

since $\frac{6^n}{5^{n-1}} > \frac{6^n}{5^n}$ AND we know that

$\sum_{n=1}^{\infty} \frac{6^n}{5^n} = \sum_{n=1}^{\infty} \left(\frac{6}{5}\right)^n$ diverges since $r = \frac{6}{5}$ and $|r| \geq 1$,

then $\sum_{n=1}^{\infty} \frac{6^n}{5^{n-1}}$ also diverges by comparison test

9. $\sum_{k=1}^{\infty} \frac{\ln k}{k} = 0 + \frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \ldots = \sum_{k=2}^{\infty} \frac{\ln k}{k}$

since the function $y = \frac{\ln x}{x}$ is continuous, positive, & decreasing on $[2, \infty)$, the integral test applies

$\int_2^{\infty} \frac{\ln x}{x}\, dx$ $\quad u = \ln x$, $du = \frac{1}{x}\,dx$ $\quad \lim_{t \to \infty} \int_{\ln 2}^{\ln t} u\, du = \frac{1}{2} u^2 \Big]_{\ln 2}^{\ln t}$

$= \frac{1}{2}(\ln t)^2 - \frac{1}{2} \ln(\ln 2)^2$

$= \frac{1}{2}(\ln \infty)^2 - \frac{1}{2} \ln(\ln(2))^2 = \text{diverges}$

since integral diverges, series diverges

10. compare $\frac{K\sin^2 K}{1+K^3}$ and $\frac{K}{1+K^3}$ and $\frac{K}{K^3}$ and $\frac{1}{K^2}$

since $\frac{K\sin^2 K}{1+K^3} < \frac{K}{1+K^3} < \frac{K}{K^3} \leq \frac{1}{K^2}$ AND we know that

$\sum_{n=1}^{\infty} \frac{1}{K^2}$ converges since p-series w/ $p>1$,

then $\sum_{n=1}^{\infty} \frac{K\sin^2 K}{1+K^3}$ also converges by comparison test

11. compare $\frac{\sqrt[3]{K}}{\sqrt{K^3+4K+3}}$ and $\frac{\sqrt[3]{K}}{\sqrt{K^3}}$

since $\frac{\sqrt[3]{K}}{\sqrt{K^3+4K+3}} < \frac{\sqrt[3]{K}}{\sqrt{K^3}}$ AND we know that

$\sum_{n=1}^{\infty} \frac{\sqrt[3]{K}}{\sqrt{K^3}} = \sum_{n=1}^{\infty} \frac{K^{1/3}}{K^{3/2}} = \sum_{n=1}^{\infty} \frac{1}{K^{7/6}}$ converges since $p>1$ by p-series test

Therefore $\sum \frac{\sqrt[3]{K}}{\sqrt{K^3+4K+3}}$ converges by comparison test

12. compare $\sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$ and $\frac{2k^3}{k^5}$

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since $\frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2} < \frac{2k^3}{k^5}$ AND we know that

$\sum_{n=1}^{\infty} \frac{2k^3}{k^5} = \sum_{n=1}^{\infty} \frac{2}{k^2}$ converges by p-series test since $p>1$,

Therefore $\sum_{n=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}$ converges by comparison test