Question 1

Let R be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the vertical line $x = -1$.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the y-axis are squares. Find the volume of this solid.

The graphs of $y = \sqrt{x}$ and $y = \frac{x}{3}$ intersect at the points $(0, 0)$ and $(9, 3)$.

(a) $\int_0^9 \left(\sqrt{x} - \frac{x}{3}\right) dx = 4.5$

OR

$\int_0^3 \left(3y - y^2\right) dy = 4.5$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\pi \int_0^3 \left((3y + 1)^2 - \left(y^2 + 1\right)^2\right) dy$

$= \frac{207\pi}{5} = 130.061$ or 130.062

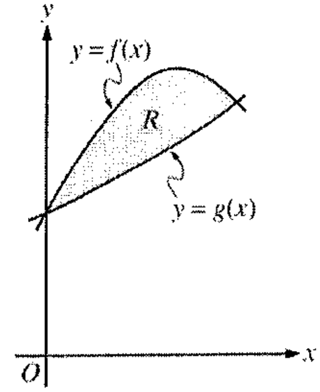
$4 : \begin{cases} 1 : \text{constant and limits} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\int_0^3 \left(3y - y^2\right)^2 dy = 8.1$

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

Question 1

Let f and g be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let R be the shaded region in the first quadrant enclosed by the graphs of f and g as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

The graphs of f and g intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$.	1 : correct limits in an integral in (a), (b), or (c)
(a) Area $= \int_0^S (f(x) - g(x))\,dx$ $= \int_0^S \left(1 + \sin(2x) - e^{x/2}\right) dx$ $= 0.429$	$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$
(b) Volume $= \pi \int_0^S \left((f(x))^2 - (g(x))^2\right) dx$ $= \pi \int_0^S \left((1 + \sin(2x))^2 - \left(e^{x/2}\right)^2\right) dx$ $= 4.266$ or 4.267	$3 : \begin{cases} 2 : \text{integrand} \\ \quad \langle -1 \rangle \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if integral not of form} \\ \quad\quad c\int_a^b \left(R^2(x) - r^2(x)\right) dx \\ 1 : \text{answer} \end{cases}$
(c) Volume $= \int_0^S \frac{\pi}{2}\left(\frac{f(x) - g(x)}{2}\right)^2 dx$ $= \int_0^S \frac{\pi}{2}\left(\frac{1 + \sin(2x) - e^{x/2}}{2}\right)^2 dx$ $= 0.077$ or 0.078	$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

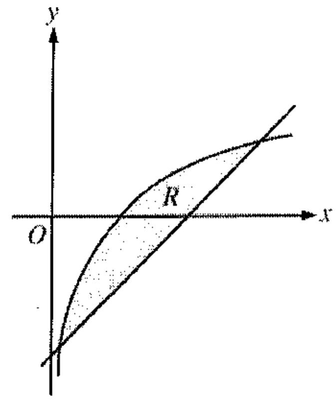
Question 1

Let R be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when R is rotated about the y-axis.



$\ln(x) = x - 2$ when $x = 0.15859$ and 3.14619.
Let $S = 0.15859$ and $T = 3.14619$

(a) Area of $R = \int_S^T (\ln(x) - (x - 2))\, dx = 1.949$

$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_S^T \left((\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx$
$= 34.198$ or 34.199

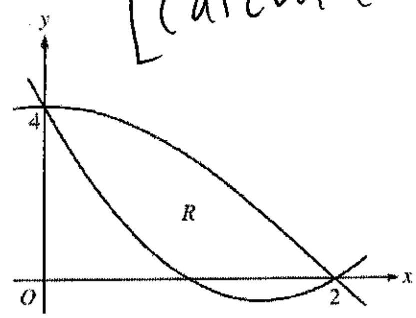
$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits, constant, and answer} \end{cases}$

(c) Volume $= \pi \int_{S-2}^{T-2} \left((y + 2)^2 - \left(e^y\right)^2 \right) dy$

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{limits and constant} \end{cases}$

no calculator

Question 5



Let $f(x) = 2x^2 - 6x + 4$ and $g(x) = 4\cos\left(\frac{1}{4}\pi x\right)$. Let R be the region bounded by the graphs of f and g, as shown in the figure above.

(a) Find the area of R.

(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 4$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an integral expression that gives the volume of the solid.

(a) $$\text{Area} = \int_0^2 [g(x) - f(x)]\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right] dx$$
$$= \left[4 \cdot \frac{4}{\pi}\sin\left(\frac{\pi}{4}x\right) - \left(\frac{2x^3}{3} - 3x^2 + 4x\right)\right]_0^2$$
$$= \frac{16}{\pi} - \left(\frac{16}{3} - 12 + 8\right) = \frac{16}{\pi} - \frac{4}{3}$$

4 : { 1 : integrand; 2 : antiderivative; 1 : answer }

(b) $$\text{Volume} = \pi\int_0^2 \left[(4 - f(x))^2 - (4 - g(x))^2\right] dx$$
$$= \pi\int_0^2 \left[\left(4 - \left(2x^2 - 6x + 4\right)\right)^2 - \left(4 - 4\cos\left(\frac{\pi}{4}x\right)\right)^2\right] dx$$

3 : { 2 : integrand; 1 : limits and constant }

(c) $$\text{Volume} = \int_0^2 [g(x) - f(x)]^2\,dx$$
$$= \int_0^2 \left[4\cos\left(\frac{\pi}{4}x\right) - \left(2x^2 - 6x + 4\right)\right]^2 dx$$

2 : { 1 : integrand; 1 : limits and constant }