11.4

1. a) You can conclude nothing about a_n because a_n could be convergent or divergent
b.) a_n is convergent by the comparison test

2. a) a_n is divergent by the comparison test
b.) We cannot conclude anything about a_n.

3. $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

$$\frac{n}{2n^3+1} < \frac{n}{2n^3} = \frac{1}{2n^2} \leftarrow$$ p-series $p>1$ so it converges

so $\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$ must converge by the comparison test.

5. $\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} + \frac{1}{n^{3/2}}$

diverges p-series $p \leq 1$

or

$$\frac{n+1}{n\sqrt{n}} > \frac{n}{n\sqrt{n}} = \frac{1}{\sqrt{n}} \leftarrow$$ diverges p-series $p \leq 1$

$\sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}$ diverges

oops 10 on next page

9. $\sum_{k=1}^{\infty} \frac{\ln k}{k}$ $\quad \frac{\ln k}{k} > \frac{1}{k}$ $\quad \frac{1}{k}$ diverges p series $p \leq 1$

$k > 3$

7. $\sum_{n=1}^{\infty} \frac{9^n}{1+10^n}$

$$\frac{9^n}{1+10^n} < \frac{9^n}{10^n} \quad n \geq 1$$

$\left(\frac{9}{10}\right)^n$ ← geometric series with $r<1$, so converges

$\sum_{n=1}^{\infty} \frac{9^n}{1+10^n}$ Converges by comparison test

11. $\sum_{n=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3+4k+3}} < \frac{\sqrt[3]{k}}{\sqrt{k^3}} \quad \frac{k^{1/3}}{k^{3/2}} = \frac{1}{k^{7/6}}$

↑ p-series $p>1$ so converges

so $\sum_{n=1}^{\infty} \frac{\sqrt[3]{k}}{\sqrt{k^3+4x+3}}$ converges by the comparison test

13. $\sum_{n=1}^{\infty} \frac{\arctan n}{n^{1.2}} \qquad \frac{\arctan n}{n^{1.2}} < \frac{\pi/2}{n^{1.2}}$

↑ p-series $p>1$ so converges

$\sum_{n=1}^{\infty} \frac{\arctan n}{n^{1.2}}$ converges by the comparison test

15. $\sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n-2}$ $\quad \frac{4(4)^n}{3^n-2} \to \left(\frac{4}{3}\right)^n$

geometric series

17. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}} = a_n \quad b_n = \frac{1}{n}$ ← diverges

$\lim_{n\to\infty} \frac{\frac{1}{\sqrt{n^2+1}}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2+1}} = 1$ so $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

diverges

19. $\sum_{n=1}^{\infty} \frac{1+4^n}{1+3^n}$ use limit comparison test

$\lim_{n\to\infty} \frac{\frac{4^n}{3^n}}{\frac{1+4^n}{1+3^n}}$ ← divergent $= \lim_{n\to\infty} \frac{(1+3^n)\cdot 4^n}{(1+4^n)\cdot 3^n}$

$= \lim_{n\to\infty} \frac{1+3^n}{3^n} \cdot \frac{4^n}{1+4^n} =$

$\lim_{n\to\infty} \left(\frac{1}{3^n}+1\right)\left(\frac{1}{\frac{1}{4^n}+1}\right) = 1\cdot 1 = 1$

so diverges

21. $\sum_{n=1}^{\infty} \frac{\sqrt{n+2}}{2n^2+n+1}$ $\quad b_n = \frac{1}{2n^{3/2}}$ ← p-series $p>1$

$\lim \sqrt{n+2}$

22. $a_n = \frac{n+2}{(n+1)^3}$ $b_n = \frac{1}{n^2}$ ← p-series $p>1$

$\lim_{n\to\infty} \frac{n^2(n+2)}{(n+1)^3} = \frac{n^3+2n^2}{n^3+\ldots} = 1$ so

by limit comparison test $\sum_{n=1}^{\infty} \frac{n+2}{(n+1)^3}$ converges

23. $\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$ $b_n = \frac{1}{n^3}$ ← p-series $p>1$

$\lim_{n\to\infty} \frac{5n^3+2n^4}{1+2n^2+n^4} = 2$ so by limit comparison test $\sum_{n=1}^{\infty} \frac{5+2n}{(1+n^2)^2}$ converges

25. $\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^3+n^2}$ $\frac{1}{n}$

$\frac{1}{n} < \frac{\sqrt{n^4+1}}{n^3+n^2}$

$\frac{\sqrt{n^4+1}}{n^2(n+1)} > \frac{n^2}{n^2(n+1)} = \frac{1}{n+1}$ = divergent

27. $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$ $\quad b_n = e^{-n} \leftarrow$ geometric $r<1$

$$\lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^2 e^{-n}}{e^{-n}} = \lim_{n\to\infty} \left[1+\frac{1}{n}\right]^2 = 1$$

so $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$ converges by limit comparison test

29. $\sum_{n=1}^{\infty} \frac{1}{n!}$ $\quad \left(\frac{1}{n}\cdot\frac{1}{(n-1)}\cdot\frac{1}{(n-2)}\cdot\frac{1}{(n-3)}\right)$

geometric series $r<1$

$$\frac{1}{n!} < \frac{1}{2^{n-1}}$$

so converges by comparison test

31. $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

$$\lim_{n\to\infty} \frac{\sin\left(\frac{1}{n}\right)}{\frac{1}{n}} = 1$$

harmonic diverges

so by limit comparison

$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$ diverges

33. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$ $\quad \frac{1}{\sqrt{n^4+1}} < \frac{1}{n^2}$

$$\sum^{10} \frac{1}{\sqrt{n^4+1}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{17}} + \cdots + \frac{1}{\sqrt{10{,}001}} \approx 1.24856$$

34. $\sum_{n=1}^{10} \frac{\sin^2 n}{n^3} \approx .83253$

$\frac{\sin^2 n}{n^3} \le \frac{1}{n^3}$

$\int_{10}^{\infty} \frac{1}{n^3}\, dn = .005$

35. $\sum_{n=1}^{10} \frac{\cos^2 n}{5^n} \approx .07393$

$\frac{\cos^2 n}{5^n} \le \frac{1}{5^n}$

$\int_{10}^{\infty} 5^{-x}\, dx \approx 6.4 \times 10^{-8}$

36. $\sum_{n=1}^{10} \frac{1}{3^n + 4^n} \approx .19788$

$\frac{1}{3^n+4^n} < \frac{1}{2(3^n)}$

$\int_{10}^{\infty} \frac{1}{2 \cdot 3^x}\, dx \approx 7.7 \times 10^{-6}$