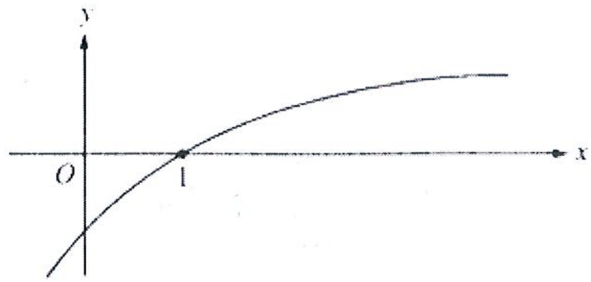

17. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f(1) < f'(1) < f''(1)$

(B) $f(1) < f''(1) < f'(1)$

(C) $f'(1) < f(1) < f''(1)$

$\boxed{\text{(D)}}$ $f''(1) < f(1) < f'(1)$

~~(E)~~ $f''(1) < f'(1) < f(1)$

Facts:

$f(1) = 0$ [function value]

$f'(1) > 0$ [positive slope]

$f''(1) < 0$ [concave down]

15. $\int x \cos x \, dx =$

(A) $x \sin x - \cos x + C$

$\boxed{\text{(B)}}$ $x \sin x + \cos x + C$

(C) $-x \sin x + \cos x + C$

(D) $x \sin x + C$

(E) $\frac{1}{2} x^2 \sin x + C$

$u = x$ $\quad$ $v = \sin x$

$du = 1\,dx$ $\quad$ $dv = \cos x \, dx$

$= x \sin x - \int \sin x \, dx$

$= x \sin x + \cos x + C$

18. Which of the following series converge?

$\lim_{n\to\infty} \frac{n}{n+2} \neq 0$ $\quad$ lim $\quad$ $\cos(\pi) = -1$, $\cos(2\pi) = 1$, $\cos(3\pi) = -1$ $\quad$ harmonic series

~~I.~~ $\sum_{n=1}^{\infty} \frac{n}{n+2}$ $\qquad$ II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ $\qquad$ ~~III.~~ $\sum_{n=1}^{\infty} \frac{1}{n}$

$\sum \frac{(-1)^n}{n}$

(A) None
$\boxed{\text{(B)}}$ II only
(C) III only
(D) I and II only
(E) I and III only

25. The closed interval $[a,b]$ is partitioned into n equal subintervals, each of width Δx, by the numbers $x_0, x_1, \ldots, x_n$ where $a = x_0 < x_1 < x_2 < \cdots < x_{n-1} < x_n = b$. What is $\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x$?

$\boxed{\text{(A)}}$ $\frac{2}{3}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(B) $b^{\frac{3}{2}} - a^{\frac{3}{2}}$

(C) $\frac{3}{2}\left(b^{\frac{3}{2}} - a^{\frac{3}{2}}\right)$

(D) $b^{\frac{1}{2}} - a^{\frac{1}{2}}$

(E) $2\left(b^{\frac{1}{2}} - a^{\frac{1}{2}}\right)$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{x_i}\,\Delta x = \int_a^b x^{1/2}\,dx$$
$$= \frac{2}{3} x^{3/2} \Big]_a^b$$
$$= \frac{2}{3} b^{3/2} - \frac{2}{3} a^{3/2}$$
$$= \frac{2}{3}\left(b^{3/2} - a^{3/2}\right)$$

76. Which of the following sequences converge?

I. $\left\{\frac{5n}{2n-1}\right\}$ $\quad \lim_{n\to\infty} \frac{5n}{2n-1} \to \frac{5}{2}$

II. $\left\{\frac{e^n}{n}\right\}$ $\quad \lim_{n\to\infty} \frac{e^n}{n} \to \frac{e^n}{1} = \infty$

III. $\left\{\frac{e^n}{1+e^n}\right\}$ $\quad \lim_{n\to\infty} \frac{e^n}{1+e^n} \Rightarrow \frac{e^n}{e^n} = 1$

(A) I only (B) II only (C) I and II only $\boxed{\text{(D)}}$ I and III only (E) I, II, and III

28) Which of the following series converge(s)?

$$\left[A = \sum_{n=1}^{\infty} \frac{1}{n^{(3/4)}},\ B = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}},\ C = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}\right]$$

$p = 3/4 \le 1$ $\quad$ $p = \frac{3}{2} > 1$ $\quad$ alt. series test

$\frac{1}{\sqrt{n+1}} \le \frac{1}{\sqrt{n}}$

a) B only
b) A, B and C
c) B and C
d) A and B
e) A and C

27) Determine

$$\int \frac{x}{x^2 + 2x - 8}\, dx$$

$$\frac{x}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2}$$

a) $\frac{1}{2} \ln((x-2)(x+4)) + C$

b) $\frac{1}{3} \ln(|x-2|) + \frac{2}{3} \ln(|x+4|) + C$

c) $-\frac{2}{3} \ln(|x-2|) - \frac{1}{3} \ln(|x+4|) + C$

d) $-\frac{2}{3} \ln(|x+4|) - \frac{1}{3} \ln(|x-2|) + C$

$x = A(x-2) + B(x+4)$

$x = Ax - 2A + Bx + 4B$

$x = \underbrace{(A+B)}_{1}x \underbrace{-2A + 4B}_{0}$

$A + B = 1$

$-2A + 4B = 0$

$= \int \frac{2/3}{x+4} + \frac{1/3}{x-2}\, dx$

$= \frac{2}{3}\ln|x+4| + \frac{1}{3}\ln|x-2| + C$

24) Determine

$$\lim_{x \to 0} \left(\frac{e^x + e^{(-x)} - 2}{1 - \cos(x)}\right)$$

a) 0
b) 1
c) 2
d) *undefined*

$\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x} \to \lim_{x \to 0} \frac{e^x + e^{-x}}{\cos x}$

$= \frac{e^0 + e^0}{\cos 0}$

$= \frac{1+1}{1} = 2$

20) Compute

$$\int e^{(3x)} (\tan(e^{(3x)}))^2 \, dx$$

u , $\frac{1}{3} du$

$= \frac{1}{3} \int \tan^2 u \, du$

$= \frac{1}{3} \int (\sec^2 u) - 1 \, du$

$= \frac{1}{3} [\tan u - u] + c$

$= \frac{1}{3} \tan e^{3x} - \frac{1}{3} e^{3x} + c$

$u = e^{3x}$

$du = 3e^{3x}$

$\frac{1}{3} du = e^{3x}$

a) $\frac{1}{3} \tan(e^{(3x)}) \, e^{(3x)} + C$

b) $\frac{1}{3} \tan(e^{(3x)}) + \frac{1}{3} e^{(3x)} + C$

c) $\frac{1}{3} \tan(e^{(3x)}) - \frac{1}{3} e^{(3x)} + C$

d) $6 \tan(e^{(3x)}) \, (\sec(e^{(3x)}))^2 \, e^{(3x)} + C$

e) $\frac{1}{3} (\sec(e^{(3x)}))^2 + C$

44. Which of the following series converge?

✓ I. $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+1}$ $\quad \frac{1}{2(n+1)+1} \le \frac{1}{2n+1}$ $\quad \lim_{n \to \infty} \frac{1}{2n+1} = 0$

$\frac{1}{2n+3} \le \frac{1}{2n+1}$

II. $\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{3}{2}\right)^n$

X III. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ positive decreasing continuous $\lim_{t \to \infty} \int_2^t \frac{1}{n \ln n} dn = \frac{1}{u} du = \ln|u| \Big]_2^t = \ln t - \ln 2 = \ln \infty$

$u = \ln n$

$du = \frac{1}{n} dn$

(A) I only
~~(B) II only~~
~~(C) III only~~
~~(D) I and III only~~
~~(E) I, II, and III~~