Which of the series converge absolutely, which converge, and which diverge? Give reasons for your answers.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}(0.1)^n$

abs. converges

by Test for absolute convergence:

$\sum (0.1)^n$ converges

since $r = \left|\frac{1}{10}\right| < 1$

2. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$

Cond. converges

since converges by Alternating Series test: ① $\frac{1}{\sqrt{n+2}} \le \frac{1}{\sqrt{n}}$ for all n ✓

② $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$ ✓

3. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n}{n^3+1}$

abs. converge

by Test for absolute convergence/comparison test

$\frac{n}{n^3+1} < \frac{n}{n^3} < \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ converges by p-series w/ p>1

4. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n+3}$

cond. converges

by Alternating series test:

① $\frac{1}{n+4} \le \frac{1}{n+3}$ ✓

② $\lim_{n\to\infty} \frac{1}{n+3} = 0$ ✓

5. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{3+n}{5+n}$

① $\frac{3+n+1}{5+n+1} \le \frac{3+n}{5+n}$

X $\frac{4+n}{6+n} \le \frac{3+n}{5+n}$

② $\lim_{n\to\infty} \frac{3+n}{5+n} = 1 \neq 0$

Diverges → Test for DIVERGENCE

7. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1+n}{n^2}$

conditionally converges

by Alternating Series Test

series test: ① $\frac{2+n}{(n+1)^2} \le \frac{1+n}{n^2}$ ✓

② $\lim_{n\to\infty} \frac{1+n}{n^2} = 0$ ✓

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+2n+1}$

abs. converge

by Test for absolute convergence/comparison test

$\frac{1}{n^2+2n+1} < \frac{1}{n^2}$

$\sum \frac{1}{n^2}$ converges by p-series w/ p>1

10. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n\sqrt{n}}$ ← note: alternating part

abs. conv.

since converges by Test for abs. conv:

$\sum \frac{1}{n\sqrt{n}} = \sum \frac{1}{n^{3/2}}$ converges

by p-series since p>1