

Estimate the magnitude of the error involved in using the sum of the first four terms to approximate the sum of the entire series.

$$11. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$$

It can be shown that the sum is $\ln 2$.

Actual sum = $\ln 2$

partial sum

$$S_4 = \frac{7}{12} = .58\overline{33}$$

$$\text{error} = \left| \ln 2 - \frac{7}{12} \right| = \boxed{0.1098}$$

* note $a_5 = \frac{1}{5} = 0.2$

$$12. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{10^n} = \frac{1}{10} - \frac{1}{100} + \frac{1}{1000} - \frac{1}{10000} + \dots + (-1)^{n+1} \cdot \frac{1}{10^n}$$

Actual sum = $\frac{a}{1-r} = \frac{\frac{1}{10}}{1 - \frac{1}{10}} = \frac{\frac{1}{10}}{\frac{9}{10}} = \frac{1}{10} \cdot \frac{10}{9} = \frac{1}{9}$

partial sum

$$S_4 = 0.0909$$

$$\text{error} = \left| \frac{1}{9} - .0909 \right| = .00000909 = \boxed{9.09 \times 10^{-6}}$$

* note: $a_5 = .00001 = 1 \times 10^{-5}$

$$13. \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(0.01)^n}{n}$$

The sum is $\ln(1.01)$

Actual sum = $\ln(1.01)$

partial sum

$$S_4 = .00995$$

$$\text{error} = \left| \ln(1.01) - 0.00995 \right| = \boxed{1.98 \times 10^{-11}}$$

* note: $a_5 = 2 \times 10^{-11}$