

$$1. y = \pm \sqrt{2(\frac{1}{2}x^2 + 3/2)}$$

$$2. y = \pm \sqrt{2(-\frac{1}{2}x^2 + 25/2)}$$

$$3. y = x$$

$$4. y = e^{x^2 + \ln 3} \quad \text{OR} \quad y = 3e^{x^2}$$

$$5. y = e^{\frac{1}{2}x^2 + 2x + \ln 6} \quad \text{OR} \quad y = 6e^{\frac{1}{2}x^2 + 2x}$$

$$6. y = \arctan x$$

$$7. y = -\ln(-e^{\sin x} - 2)$$

$$8. y = \ln(e^x + 1)$$

$$9. y = \frac{1}{x^2 + 3}$$

$$10. y = (\ln x)^4$$

$$11. y = 100e^{1.5t} \quad \text{OR} \quad y = e^{1.5t + \ln 100}$$

$$12. y = 200e^{-0.5t} \quad \text{OR} \quad y = e^{-0.5t + \ln 200}$$

$$13. y = 50e^{kt} \quad \text{AND} \quad y = e^{kt + \ln 100 - 5k}$$

$$14. y = 60e^{kt} \quad \text{AND} \quad y = e^{kt + \ln 30 - 10k}$$

Section 6.4 Exercises

In Exercises 1–10, use separation of variables to solve the initial value problem. Indicate the domain over which the solution is valid.

$$1. \frac{dy}{dx} = \frac{x}{y} \quad \text{and } y = 2 \text{ when } x = 1$$

$$2. \frac{dy}{dx} = -\frac{x}{y} \quad \text{and } y = 3 \text{ when } x = 4$$

$$3. \frac{dy}{dx} = \frac{y}{x} \quad \text{and } y = 2 \text{ when } x = 2$$

$$4. \frac{dy}{dx} = 2xy \quad \text{and } y = 3 \text{ when } x = 0$$

$$5. \frac{dy}{dx} = (y+5)(x+2) \quad \text{and } y = 1 \text{ when } x = 0$$

$$6. \frac{dy}{dx} = \cos^2 y \quad \text{and } y = 0 \text{ when } x = 0$$

$$7. \frac{dy}{dx} = (\cos x)e^{y+\sin x} \quad \text{and } y = 0 \text{ when } x = 0$$

$$8. \frac{dy}{dx} = e^{x-y} \quad \text{and } y = 2 \text{ when } x = 0$$

$$9. \frac{dy}{dx} = -2xy^2 \quad \text{and } y = 0.25 \text{ when } x = 1$$

$$10. \frac{dy}{dx} = \frac{4\sqrt{y} \ln x}{x} \quad \text{and } y = 1 \text{ when } x = e$$

In Exercises 11–14, find the solution of the differential equation $dy/dt = ky$, k a constant, that satisfies the given conditions.

$$11. k = 1.5, \quad y(0) = 100 \qquad 12. k = -0.5, \quad y(0) = 200$$

$$13. y(0) = 50, \quad y(5) = 100 \qquad 14. y(0) = 60, \quad y(10) = 30$$

#1-6 in class

$$\#7. \frac{dy}{dx} = (\cos x) e^{y+\sin x}$$

$$\frac{dy}{dx} = \cos x \cdot e^y \cdot e^{\sin x}$$

$$\frac{1}{e^y} dy = \cos x \cdot e^{\sin x} dx$$

$$\int e^{-y} dy = \int \boxed{\cos x} e^{\boxed{\sin x}} \boxed{dx}$$

$$\begin{aligned} u &= \sin x \\ \frac{du}{dx} &= \cos x \\ du &= \cos x dx \end{aligned}$$

$$\int e^{-y} dy = \int e^u du$$

$$-e^{-y} = e^u + c$$

$$-e^{-y} = e^{\sin x} + c$$

$$e^{-y} = -(e^{\sin x} + c)$$

$$\ln(e^{-y}) = \ln(-(e^{\sin x} + c))$$

$$-y = \ln(-(e^{\sin x} + c))$$

$$\boxed{y = -\ln(-(e^{\sin x} + c))}$$

general solution

$$0 = -\ln(-(e^{\sin 0} + c))$$

$$0 = -\ln(-(e^0 + c))$$

$$0 = -\ln(-(1+c))$$

$$0 = \ln(-(1+c))$$

e e

$$1 = -(1+c)$$

$$-1 = 1+c$$

$$c = -2$$

particular solutions

$$\boxed{y = -\ln(-(e^{\sin x} - 2))}$$

$$\#8 \frac{dy}{dx} = e^{x-y}$$

$$\frac{dy}{dx} = e^x \cdot e^{-y}$$

$$\frac{1}{e^{-y}} dy = e^x dx$$

$$\int e^y dy = \int e^x dx$$

$$e^y = e^x + c$$

$$\ln(e^y) = \ln(e^x + c)$$

$$\boxed{y = \ln(e^x + c)}$$

general solution

$$2 = \ln(e^0 + c)$$

$$2 = \ln(1+c)$$

$$e^2 = e^{\ln(1+c)}$$

$$2 = \cancel{1+c} 1+c$$

$$c = 1$$

$$\boxed{y = \ln(e^x + 1)}$$

particular solution

$$\# 9 \quad \frac{dy}{dx} = -2xy^2$$

$$\frac{1}{y^2} dy = -2x dx$$

$$\int y^{-2} dy = \int -2x dx$$

$$-y^{-1} = -x^2 + c$$

$$y^{-1} = x^2 - c$$

$$\frac{1}{y} = x^2 - c$$

$$\boxed{y = \frac{1}{x^2 - c}}$$

general solution

$$0.25 = \frac{1}{(1)^2 - c}$$

$$0.25(1 - c) = 1$$

$$1 - c = \frac{1}{0.25}$$

$$1 - c = 4$$

$$c = -3$$

$$\boxed{y = \frac{1}{x^2 + 3}}$$

particular sol.

$$\# 10 \quad \frac{dy}{dx} = \frac{4\sqrt{y} \ln x}{x}$$

$$\frac{1}{\sqrt{y}} dy = \frac{4 \ln x}{x} dx$$

$$\int y^{-1/2} dy = \int 4 \frac{\ln x}{x} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$\int y^{-1/2} dy = \int 4u du$$

$$2y^{1/2} = 2u^2 + c$$

$$2y^{1/2} = 2(\ln x)^2 + c$$

$$y^{1/2} = \frac{1}{2} (2(\ln x)^2 + c)$$

$$(y^{1/2})^2 = \left(\frac{1}{2} (2(\ln x)^2 + c) \right)^2$$

$$\boxed{y = ((\ln x)^2 + \frac{1}{2}c)^2}$$

general sol.

$$1 = ((\ln e)^2 + \frac{1}{2}c)^2$$

$$1 = (1 + \frac{1}{2}c)^2$$

$$\sqrt{1} = \sqrt{(1 + \frac{1}{2}c)^2}$$

$$1 = 1 + \frac{1}{2}c$$

$$c = 0$$

$$\boxed{y = [(\ln x)^2]^2} = \boxed{(\ln x)^4}$$

particular sol.

$$\boxed{\frac{dy}{dt} = ky}$$

11. $\frac{dy}{dt} = 1.5y$

$$\int \frac{1}{y} dy = \int 1.5 dt$$

$$e^{\ln|y|} = e^{1.5t + c}$$

$$\boxed{y = e^{1.5t + c}}$$

gen. solution

$$100 = e^{1.5(0) + c}$$

$$100 = e^c$$

$$c = \ln(100)$$

OR $\left\{ \boxed{y = e^{1.5t + \ln 100}} \right.$
exact solution

$$\boxed{y = 100e^{1.5t}}$$

#12 $\frac{dy}{dt} = -0.5y$

$$\int \frac{1}{y} dy = \int -0.5 dt$$

$$e^{\ln|y|} = e^{-0.5t + c}$$

$$\boxed{y = e^{-0.5t + c}}$$

gen. sol.

$$200 = e^{-0.5(0) + c}$$

$$200 = e^c$$

$$c = \ln(200)$$

OR $\left\{ \boxed{y = e^{-0.5t + \ln(200)}} \right.$
exact sol.
 $\boxed{y = 200e^{-0.5t}}$

#13

$$\frac{dy}{dt} = ky$$

$$\frac{1}{y} = k dt$$

$$\ln|y| = kt + c$$

$$\boxed{y = e^{kt + c}} \text{ gen. sol.}$$

(0, 50)

$$50 = e^{0 + c}$$

$$c = \ln 50$$

$$\boxed{y = e^{kt + \ln 50}}$$

OR $\boxed{y = 50e^{kt}}$

(5, 100)

$$100 = e^{5k + c}$$

$$\ln 100 = 5k + c$$

$$c = \ln(100) - 5k$$

$$\boxed{y = e^{kt + \ln(100) - 5k}}$$

#14

$$\frac{dy}{dt} = ky$$

$$\boxed{y = e^{kt + c}} \text{ gen. sol.}$$

(0, 60)

$$60 = e^{k(0) + c}$$

$$60 = e^{0 + c}$$

$$60 = e^c$$

$$c = \ln 60$$

$$\boxed{y = e^{kt + \ln 60}}$$

OR $\boxed{y = 60e^{kt}}$

(10, 30)

$$30 = e^{k(10) + c}$$

$$30 = e^{10k + c}$$

$$\ln 30 = 10k + c$$

$$c = \ln(30) - 10k$$

$$\boxed{y = e^{kt + \ln(30) - 10k}}$$