

1. Determine whether the following series converge or diverge. State which convergence test you used.

A.  $\sum_{n=1}^{\infty} (\frac{1}{2})^n$  CONVERGE

geometric series w/  $|\frac{1}{2}| < 1$

B.  $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\sqrt{n-1}}$  =  $\frac{1}{\sqrt{n^2-n^2}} = \frac{1}{(n^2-n^2)^{1/4}}$  DIVERGE

comparison test:  $\frac{1}{(n^2-n^2)^{1/4}} > \frac{1}{(n^2)^{1/4}}$

since  $\sum \frac{1}{(n^2)^{1/4}}$  diverges by p-series with  $p < 1$  then given series diverges

C.  $\sum_{n=1}^{\infty} \frac{1}{2n+1}$  DIVERGE

~~comparison test~~ OR integral Test

note  $y = \frac{1}{2x+1}$  is cont, pos, dec on  $[1, \infty)$  so:

$\lim_{t \rightarrow \infty} \int_1^t \frac{1}{2x+1} dx = \frac{1}{2} \ln|2x+1| \Big|_1^t = \frac{1}{2} \ln|2t+1| - \frac{1}{2} \ln 3 = \infty$

D.  $\sum_{n=0}^{\infty} \left(-\frac{1}{3}\right)^n$  CONVERGE

Geometric series w/  $|\frac{-1}{3}| < 1$

E.  $\sum_{n=1}^{\infty} \frac{(-1)^n n^3}{n^2}$  DIVERGE

Test for divergence

$\lim_{n \rightarrow \infty} a_n \neq 0$

F.  $\sum_{n=0}^{\infty} 2^n$  DIVERGE

Geometric series w/  $|2| \geq 1$

G.  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$  CONVERGE

alternating series test

①  $\frac{1}{\sqrt{n+1}} \leq \frac{1}{\sqrt{n}}$  for all n ✓

②  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$  ✓

\*NOTE: WHY conditionally converges

2. Suppose you know that  $0 \leq b_n \leq \frac{1}{n} \leq a_n$  and  $0 \leq c_n \leq \frac{1}{n^2} \leq d_n$  for all  $n$ .

A. Which of the series  $\sum a_n$ ,  $\sum b_n$ ,  $\sum c_n$ , and  $\sum d_n$  definitely converge? Justify your answer.

$\sum c_n$  converges

comparison test!

if  $c_n \leq \frac{1}{n^2}$

&  $\sum \frac{1}{n^2}$  converges by p-series w/  $p > 2$

then  $\sum c_n$  converges

B. Which of the series  $\sum a_n$ ,  $\sum b_n$ ,  $\sum c_n$ , and  $\sum d_n$  definitely diverge? Justify your answer.

$\sum a_n$  diverges

comparison test!

if  $a_n \geq \frac{1}{n}$

and  $\sum \frac{1}{n}$  diverges by harmonic series

then  $\sum a_n$  diverges