1. What is a power series? a series in the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \dots$ where x is a variable

2. (a) What is the radius of convergence of a power series?
How do you find it?
(b) What is the interval of convergence of a power series?
How do you find it?
-interval that consists of all values of x for which the series converges
-find it by considering what values of x make $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| < 1$

3–28 Find the radius of convergence and interval of convergence of the series.

3. $\sum_{n=1}^{\infty} (-1)^n n x^n$

4. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

5. $\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$

6. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

7. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

~~8. $\sum_{n=1}^{\infty} n^n x^n$~~

Note... 1

3. $$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(n+1)\cdot x^{n+1}}{(-1)^n n x^n} \right| = \left| \frac{\cancel{(-1)^n}\cdot(-1)\cdot(n+1)\cdot \cancel{x^n}\cdot x}{\cancel{(-1)^n}\cdot n \cdot \cancel{x^n}} \right| = \left| -x \cdot \frac{n+1}{n} \right|$$

$$= |x|$$

By ratio test, series converges when $|x| < 1$,
so radius of convergence is 1

check endpts... that is ± 1.

if $x = 1$, then $\sum (-1)^n n \to$ DIVERGES [Test for Diver.]

if $x = -1$, then $\sum (-1)^n n (-1)^n \to$
$\sum (-1)^{2n} \cdot n \to$ DIVERGES [Test for Diver.]

So interval of convergence is $(-1, 1)$

Note... 1

4. $\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{(-1)^n x^n} \right| = \left| \frac{(-1)\, x \boxed{\sqrt[3]{n}}}{\boxed{\sqrt[3]{n+1}}} \right| = |x|$

By ratio test series converges when $|x|<1$,
so radius of convergence is 1

check endpts..., that is ± 1.

if $x=1$, then $\sum \frac{(-1)^n}{\sqrt[3]{n}}$ → CONVERGES [alt. series test]

if $x=-1$, then $\sum \frac{(-1)^n (-1)^n}{\sqrt[3]{n}} = \sum \frac{(-1)^{2n}}{\sqrt[3]{n}}$ → DIVERGES [p-series]

so interval of convergence is $(-1, 1]$

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5. $\lim_{n\to\infty} \left| \frac{x^{n+1}}{2(n+1)} \cdot \frac{2n+1}{x^n} \right| = \left| x \cdot \boxed{\frac{2n+1}{2n+2}} \right| = |x|$

1

By ratio test, series converges when $|x|<1$,
So radius of convergence is 1

check endpts... that is ± 1.

if $x=1$, then $\sum \frac{1}{2n-1}$ → DIVERGES [comparison to harmonic]

if $x=-1$, then $\sum \frac{(-1)^n}{2n-1}$ → CONVERGES [alt. series test]

so interval of convergence is $[-1, 1)$

6. $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$

Note... 1

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} \cdot x^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-1)^n x^n} \right| = \left| \frac{(-1) \cdot x \cdot \boxed{n^2}}{\boxed{(n+1)^2}} \right| = |x|$$

By ratio test, series converges when $|x| < 1$, so radius of convergence is 1

check endpts... that is ± 1.

if $x = 1$, then $\sum \frac{(-1)^n \cdot \cancel{1^n}^{\,1}}{n^2} \rightarrow$ CONVERGES [alt. series test + p-series]

if $x = -1$, then $\sum \frac{(-1)^n \cdot (-1)^n}{n^2} = \sum \frac{1}{n^2} \rightarrow$ CONVERGES [p-series]

interval of convergence is $[-1, 1]$

7. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$$\lim_{n\to\infty} \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right| = \left| \frac{\cancel{x^n} \cdot x \cdot \cancel{n!}}{(n+1)\cancel{n!}\,\cancel{x^n}} \right| = \left| \frac{x}{n+1} \right|$$

Think: $\lim_{n\to\infty} \left| \frac{1}{n+1} \right| = 0$... x in numerator will not affect this

By ratio test, series con. if $\left| \frac{x}{n+1} \right| < 1$

Series converges for all values of x, $(-\infty, \infty)$