9. $\sum_{n=1}^{\infty} (-1)^n \cdot \frac{n^2 x^n}{2^n}$

$$\lim_{n\to\infty} \left| \frac{\frac{(n+1)^2 x^n \cdot x}{2^n \cdot 2}}{\frac{n^2 x^n}{2^n}} \right| = \left| \frac{x \cdot \overbrace{(n+1)^2}^{\text{limit}=1}}{2 \cdot n^2} \right| = \left| \frac{x}{2} \right| = \frac{|x|}{2}$$

By ratio test, converges if $\frac{|x|}{2} < 1$

$|x| < 2$

radius of con. is 2

check endpts: $x = -2, 2$

if $x = -2$, $\sum (-1)^n \cdot n^2 \cdot \frac{(-2)^n}{2^n} = \sum (1)^n n^2 \rightarrow$ DIVERGES

if $x = 2$, $\sum (-1)^n \cdot n^2 \cdot \frac{(2)^n}{2^n} = \sum (-1)^n n^2 \rightarrow$ DIVERGES

int. of conv is $(-2, 2)$

10. $\sum_{n=1}^{\infty} \frac{10^n \cdot x^n}{n^3}$

limit = 1

$$\lim_{n\to\infty} \left| \frac{\cancel{10^n} \cdot 10 \cdot \cancel{x^n} \cdot x}{\boxed{(n+1)^3}} \cdot \frac{\boxed{n^3}}{\cancel{10^n} \cdot \cancel{x^n}} \right| = |10x| = 10|x|$$

By ratio test, series converges if $10|x| < 1$

$|x| < \frac{1}{10}$

radius of con. is $\frac{1}{10}$

check endpts $x = \pm \frac{1}{10}$

if $x = -\frac{1}{10}$, then $\sum \frac{10^n \cdot (-\frac{1}{10})^n}{n^3} = \sum \frac{(-1)^n}{n^3} \to$ CONVERGES [alt. series test]

if $x = \frac{1}{10}$, then $\sum \frac{10^n \cdot (\frac{1}{10})^n}{n^3} = \sum \frac{1}{n^3} \to$ CONVERGES [p-series]

interval of con: $\left[-\frac{1}{10}, \frac{1}{10}\right]$

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11. $\sum_{n=1}^{\infty} \frac{(-3)^n}{n\sqrt{n}} x^n$

limit = 1

$$\lim_{n\to\infty} \left| \frac{\cancel{(-3)^n} \cdot (-3) \cdot \cancel{x^n} \cdot x}{\boxed{(n+1)\sqrt{n+1}}} \cdot \frac{\boxed{n\sqrt{n+1}}}{\cancel{(-3)^n}\, \cancel{x^n}} \right| = |-3x| = 3|x|$$

By ratio test, series con. if $3|x| < 1$

$|x| < 1/3$

rad. of conv. is 1/3

check endpts $x = \pm 1/3$

if $x = -\frac{1}{3}$, then $\sum \frac{(-3)^n \cdot (-1/3)^n}{n\sqrt{n}} = \sum \frac{1}{n^{3/2}} \to$ converges [p-series]

if $x = \frac{1}{3}$, then $\sum \frac{(-3)^n \cdot (\frac{1}{3})^n}{n\sqrt{n}} = \sum \frac{(-1)^n}{n^{3/2}} \to$ converges [alt. series]

interval of conv is $-\frac{1}{3} \le x \le \frac{1}{3}$

12. $\sum_{n=1}^{\infty} \frac{x^n}{n\,3^n}$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^n}\cdot x}{(n+1)\,\cancel{3^n}\cdot 3} \cdot \frac{n\,\cancel{3^n}}{\cancel{x^n}} \right| = \left|\tfrac{1}{3}x\right| = \tfrac{1}{3}|x|$$

limit = 1

by ratio test, series converges if $\frac{1}{3}|x| < 1$

$|x| < 3$

Radius of con. is 3

Check endpts $x = \pm 3$

if $x = -3$, then $\sum \frac{(-3)^n}{n\cdot 3^n} = \sum \frac{(-1)^n}{n} \rightarrow$ converges [alt. series test]

if $x = 3$, then $\sum \frac{3^n}{n\cdot 3^n} = \sum \frac{1}{n} \rightarrow$ diverges [harmonic]

int. of convergence is $[-3, 3)$

$\sum_{n=2}^{\infty} (-1)^n \cdot \frac{x^n}{4^n \ln n}$

$$\lim_{n\to\infty} \left| \frac{\cancel{x^n}\cdot x}{\cancel{4^n}\cdot 4\cdot \boxed{\ln(n+1)}} \cdot \frac{\cancel{4^n}\,\boxed{\ln n}}{\cancel{x^n}} \right| = \left|\frac{x}{4}\right| = \frac{1}{4}|x|$$

limit = 1

By ratio test, series converges if $\frac{|x|}{4} < 1$

so radius of con. is 4

$|x| < 4$

Check endpts $x = \pm 4$

if $x = -4$, then $\sum \frac{\cancel{(-1)^n}\cdot\cancel{(4^n)}}{\cancel{4^n}\ln n} = \sum \frac{1}{\ln n} \rightarrow$ DIVERGES [comp. to harmonic: $\frac{1}{\ln n} > \frac{1}{n}$]

if $x = 4$, then $\sum \frac{(-1)^n\cdot \cancel{4^n}}{\cancel{4^n}\ln n} = \sum \frac{(-1)^n}{\ln n} \rightarrow$ CONVERGES [alt. series test]

int. of convergence is $-4 < x \le 4$

14. $\boxed{\displaystyle\sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!}}$

$$\lim_{n\to\infty} \left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \cdot \frac{(2n+1)!}{x^{2n+1}} \right| = \left| \frac{\cancel{x^{2n}} \cdot x^3 \cdot \cancel{(2n+1)!}}{(2n+3)(2n+2)\cancel{(2n+1)!}\,\cancel{x^{2n}} \cdot x} \right| = \left| \frac{x^2}{(2n+3)(2n+2)} \right|$$

Note: $\lim\limits_{n\to\infty} \frac{1}{(2n+3)(2n+2)}$ is 0 for all values of x

By ratio test, series con. if $\left| \frac{x^2}{(2n+3)(2n+2)} \right| < 1$

$$\left| x^2 \cdot \frac{1}{(2n+3)(2n+2)} \right| < 1$$

limit = 0

Series converges for all values of x

$(-\infty, \infty)$

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23. $\displaystyle\sum_{n=1}^{\infty} n!\,(2x-1)^n$

$$\lim_{n\to\infty} \left| \frac{(n+1)!\,\cancel{(2x-1)^n} \cdot (2x-1)}{n!\,\cancel{(2x-1)^n}} \right| = \left| (n+1)(2x-1) \right| = n+1\,|2x-1|$$

$\lim\limits_{n\to\infty} n+1\,|2x-1| = \infty$

for all values of x EXCEPT $x = \frac{1}{2}$

Series converges at $x = 1/2$ only

Radius of con. is 0