3. $f(x) = \frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{n=0}^{\infty} (-x)^n = \boxed{\sum_{n=0}^{\infty} (-1)^n x^n}$

r

Converges when $|-x| < 1$

$|x| < 1$ } so radius of con. is 1

test endpts... $x = \pm 1$ ... DIVERGE

$\boxed{\text{Interval of convergence } (-1, 1)}$

4. $f(x) = \frac{5}{1-4x^2} = 5 \cdot \frac{1}{1-(4x^2)} = 5 \sum_{n=0}^{\infty} (4x^2)^n = \boxed{\sum_{n=0}^{\infty} 5 \cdot 4^n \cdot x^{2n}}$

r

converges when $|4x^2| < 1$

$|x^2| < 1/4$

$x^2 < \frac{1}{4}$ } so radius of convergence is $\frac{1}{2}$

$|x| < \frac{1}{2}$

test endpts... $x = \pm 1/2$ ... DIVERGE

$\boxed{\text{interval of convergence } (-1/2, 1/2)}$

5. $f(x) = \frac{2}{3-x} = \frac{2}{3(1-\frac{x}{3})} = \frac{2}{3} \sum \left(\frac{x}{3}\right)^n = \boxed{\sum_{n=0}^{\infty} \frac{2}{3} \cdot \frac{x^n}{3^n}} = \sum_{n=0}^{\infty} 2 \cdot \frac{1}{3^{n+1}} x^n$

r

converges when $\left|\frac{x}{3}\right| < 1$

$\frac{|x|}{3} < 1$ } so radius of convergence is 3

$|x| < 3$

test endpts... $x = \pm 3$ ... DIVERGE

$\boxed{\text{int. of convergence is } (-3, 3)}$

6. $f(x) = \frac{1}{x+10} = \frac{1}{10(1+\frac{x}{10})} = \frac{1}{10[1-(-\frac{x}{10})]} = \frac{1}{10} \cdot \frac{1}{1-(-\frac{x}{10})}$

$$\frac{1}{10} \sum \left(-\frac{x}{10}\right)^n = \boxed{\frac{1}{10} \sum_{n=0}^{\infty} (-1)^n \cdot \frac{x^n}{10^n}} = \sum \frac{(-1)^n \cdot x^n}{10 \cdot 10^n}$$

$$= \sum \frac{(-1)^n \cdot x^n}{10^{n+1}}$$

converges if $\left|-\frac{x}{10}\right| < 1$

$$\left.\begin{aligned} \frac{|x|}{10} &< 1 \\ |x| &< 10 \end{aligned}\right\}$$ so radius of convergence is 10

test endpts $x = \pm 10$ ... DIVERGE

interval of convergence $(-10, 10)$

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7. $f(x) = \frac{x}{9+x^2} = \frac{x}{9}\left[\frac{1}{(1+\frac{x^2}{9})}\right] = \frac{x}{9}\left[\frac{1}{[1-(-\frac{x^2}{9})]}\right]$

$$= \sum \frac{x}{9} \cdot \left(\frac{-x^2}{9}\right)^n = \sum \frac{x}{9} \cdot (-1)^n \cdot \frac{x^{2n}}{9^n} = \boxed{\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{9^{n+1}}}$$

converges if $\left|-\frac{x^2}{9}\right| < 1$

$$\left.\begin{aligned} |-x^2| &< 9 \\ x^2 &< 9 \\ |x| &< 3 \end{aligned}\right\}$$ so radius of convergence is 3

test endpts $x = \pm 3$ ... DIVERGE

interval of convergence: $(-3, 3)$

8. $f(x) = \dfrac{x}{2x^2+1} = x\left(\dfrac{1}{1-(-2x^2)}\right) = \sum x \cdot (-2x^2)^n = \boxed{\sum_{n=0}^{\infty} (-1)^n \cdot 2^n \cdot x^{2n+1}}$

converges if $|-2x^2| < 1$

$$\left.\begin{aligned} 2|x^2| &< 1 \\ x^2 &< 1/2 \\ |x| &< \frac{1}{\sqrt{2}} \end{aligned}\right\} \text{so radius of convergence is } \frac{1}{\sqrt{2}}$$

test endpoints: $x = \pm \frac{1}{\sqrt{2}}$ ... DIVERGE

interval of convergence: $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$f(x) = \dfrac{1+x}{1-x} = 1+x\left(\dfrac{1}{1-x}\right) = \sum (1+x) \cdot x^n = \sum_{n=1}^{\infty} x^n + x^{n+1}$

Converges if $|x| < 1$ $\Big\}$ so radius of convergence is 1

test endpts $x = \pm 1$ ... DIVERGE

interval of convergence: $(-1, 1)$