1 a. $f(x) \approx 1 + 4(x-3) + 3(x-3)^2 + 2(x-3)^3$

$f(3.2) \approx 1.936$

b. $f'(x) = 0 + 4 + 6x + 6(x-3)^2$

$f'(2.7) \approx 2.74$

c. point: $(3,1)$

slope $= 4$

tangent: $y-1 = 4(x-3)$

underestimation since $f'' > 0$

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2 a. $f(4) = 7 - 3\cancel{(4-4)} + 5\cancel{(4-4)}^2 - 2\cancel{(4-4)}^3 + 6\cancel{(4-4)}^4$

$f(4) = 7$

b. $f'(x) = -3 + 10(x-4) + -6(x-4)^2 + 24(x-4)^3$

$f''(x) = 10 - 12(x-4) + 72(x-4)^2$

$f'''(x) = -12 + 144(x-4)$

$f'''(4) = -12 + \cancel{144(4-4)}$

$f'''(4) = -12$

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3 a. $f'(x) = 2 + 4x + 8x^2 + 16x^3 + \ldots \underbrace{2(2x)^n}_{\text{general term}} + \ldots$

b. $f'(-1/3) \approx 2 + 4(-1/3) + 8(-1/3)^2 + 16(-1/3)^3 = 0.963$

appx sum:

actual sum: $\frac{2}{1-(-2/3)} = 6/5$

4 a. $f(x) = \frac{1}{1-\underbrace{2x}_{r}} = \sum_{n=0}^{\infty} (\underbrace{2x}_{r})^n = 1 + 2x + 4x^2 + 8x^3 + \dots + \underbrace{(2x)^n}_{\text{general term}} + \dots$

b. int. of convergence work:

$|2x| < 1$

$2|x| < 1$

$|x| < \frac{1}{2}$

$R = 1/2$

interval of con: $(-\frac{1}{2}, \frac{1}{2})$

c. $0.02 = \frac{1}{50}$ ← next unused term should be greater than this

$$f(-\tfrac{1}{4}) \approx \underbrace{1}_{1} + \underbrace{2(-\tfrac{1}{4})}_{-1/2} + \underbrace{4(-\tfrac{1}{4})^2}_{1/4} + \underbrace{8(-\tfrac{1}{4})^3}_{-.125} + \underbrace{16(-1/4)^4}_{.0625} + \underbrace{32(-1/4)^5}_{.03125} + \underbrace{64(-1/4)^6}_{.0156}$$

0.02

$f(-1/4) = \frac{21}{32}$

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5 a. $\lim_{n\to\infty} \left| \frac{\frac{x^{n+1}(n+1)^{n+1}}{(n+1)!}}{\frac{x^n n^n}{n!}} \right| = \left| \frac{\cancel{x^{n+1}} \cdot (n+1)^n \cdot (n+1) \cdot \cancel{n!}}{\cancel{x^n} \cdot n^n \cdot \cancel{(n+1)!}} \right|$

$x \quad e \quad 1$

$(n+1)$

$= |ex| < 1$

$= e|x| < 1$

$= |x| < \frac{1}{e} \longrightarrow$ rad. of con. $= \frac{1}{e}$

b. $f(x) = x + 2x^2 + \frac{27}{6}x^3$

$f(-1/3) = -\frac{1}{3} + \frac{2}{9} - \frac{1}{6}$

$f(-1/3) = -5/18$

c. By Alt. Series Remainder, error is no more than magnitude of the next term: $\frac{(-1/3)\cdot 4^4}{4!} \approx 0.132$