1.8

approach 0. To do this we use our knowledge of the sine function. Because the sine of any number lies between -1 and 1, we can write

$$\boxed{4} \qquad -1 \leqslant \sin\frac{1}{x} \leqslant 1$$

Any inequality remains true when multiplied by a positive number. We know that $x^2 \geqslant 0$ for all x and so, multiplying each side of the inequalities in $\boxed{4}$ by x^2, we get

$$-x^2 \leqslant x^2 \sin\frac{1}{x} \leqslant x^2$$

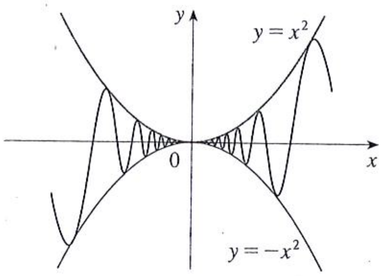


FIGURE 8
$y = x^2 \sin(1/x)$

as illustrated by Figure 8. We know that

~~(1+3)(1−5+3)~~
~~4(−)~~

$$\lim_{x\to 0} x^2 = 0 \qquad \text{and} \qquad \lim_{x\to 0}(-x^2) = 0$$

Taking $f(x) = -x^2$, $g(x) = x^2 \sin(1/x)$, and $h(x) = x^2$ in the Squeeze Theorem, we obtain

$\frac{14}{8+6+2} = \frac{14}{16}$

$$\lim_{x\to 0} x^2 \sin\frac{1}{x} = 0$$

$\frac{7}{8}$

2.3 Exercises

1. Given that

$$\lim_{x\to 2} f(x) = 4 \qquad \lim_{x\to 2} g(x) = -2 \qquad \lim_{x\to 2} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to 2}[f(x) + 5g(x)]$ $\quad 4 + 5(-2) = -6$

(b) $\lim_{x\to 2}[g(x)]^3$ $\quad (-2)^3 = -8$

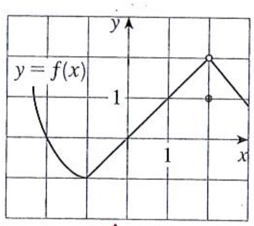
(c) $\lim_{x\to 2}\sqrt{f(x)}$ $\quad \sqrt{4} = 2$

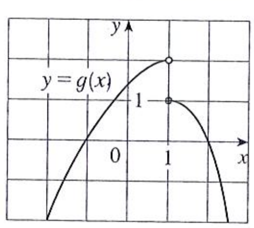
(d) $\lim_{x\to 2}\dfrac{3f(x)}{g(x)}$ $\quad \dfrac{3(4)}{-2} = -6$

(e) $\lim_{x\to 2}\dfrac{g(x)}{h(x)}$ $\quad \dfrac{-2}{0} =$ DNE undefined!

(f) $\lim_{x\to 2}\dfrac{g(x)h(x)}{f(x)}$ $\quad \dfrac{-2(0)}{4} = \boxed{0}$

2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





(a) $\lim_{x\to 2}[f(x) + g(x)]$ $\quad 2 + 0 = 2$

(b) $\lim_{x\to 1}[f(x) + g(x)]$ $\quad 1 +$ DNE $=$ DNE

(c) $\lim_{x\to 0}[f(x)g(x)]$ $\boxed{0}$ $\quad 0 \cdot 1.5 =$

(d) $\lim_{x\to -1}\dfrac{f(x)}{g(x)}$ $\quad \dfrac{-1}{-1} = 1$

(e) $\lim_{x\to 2}[x^3 f(x)]$ $\quad (2^3)2 = 16$

(f) $\lim_{x\to 1}\sqrt{3 + f(x)}$ $\quad \sqrt{3+1} = 2$

3–9 Evaluate the limit ~~and justify each step by indicating the appropriate Limit Law(s).~~

3. $\lim_{x\to 3}(5x^3 - 3x^2 + x - 6)$ $\quad 5(3)^3 - 3(3)^2 + 3 - 6 = 105$

4. $\lim_{x\to -1}(x^4 - 3x)(x^2 + 5x + 3)$ $\quad -4$

5. $\lim_{t\to -2}\dfrac{t^4 - 2}{2t^2 - 3t + 2}$ $= \dfrac{7}{8}$

6. $\lim_{u\to -2}\sqrt{u^4 + 3u + 6}$ $= 4$

7. $\lim_{x\to 8}(1 + \sqrt[3]{x})(2 - 6x^2 + x^3)$ $\quad 390$

8. $\lim_{t\to 2}\left(\dfrac{t^2 - 2}{t^3 - 3t + 5}\right)^2$ $= \dfrac{4}{49}$

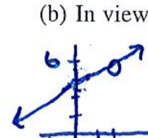
9. $\lim_{x\to 2}\sqrt{\dfrac{2x^2 + 1}{3x - 2}}$ $= \sqrt{\dfrac{2(2)^2+1}{3(2)-2}} = \dfrac{3}{2}$

10. (a) What is wrong with the following equation?

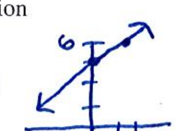
not defined at $x = 2$ $\qquad$ factor $\qquad$ defined everywhere

$$\frac{x^2 + x - 6}{x - 2} = x + 3$$

(b) In view of part (a), explain why the equation



$$\lim_{x\to 2}\frac{x^2 + x - 6}{x - 2} = \lim_{x\to 2}(x + 3)$$



is correct. there is a hole at $x = 2$ on the first graph, However the limit is still approaching 5 on either side of $x = 2$

Limits Algebraically

Find the following limits:

Substitution

1. $\lim_{x\to 2}(x^2 - x + 1)$

$(2)^2 - (2) + 1 = \boxed{3}$

2. $\lim_{x\to 1}\left(\dfrac{2x+1}{3x-2}\right)$

$= \dfrac{2+1}{3-2} = \boxed{3}$

3. $\lim_{x\to 1}(\sqrt{10x-1})$

$= \sqrt{10-1}$
$= \boxed{3}$

4. $\lim_{x\to 1}\left(\dfrac{x^2-x-2}{x-2}\right)$

$\dfrac{1-1-2}{1-2} = \boxed{2}$

5. $\lim_{x\to 2}\left(\dfrac{x^2-x-2}{4x-2}\right)$

$\dfrac{4-2-2}{8-2} = \boxed{0}$

6. $\lim_{x\to 4}\left(\dfrac{\sqrt{x}-2}{2x-4}\right)$

$= \dfrac{2-2}{8-4} = \boxed{0}$

7. $\lim_{x\to -3}\left(\dfrac{x^2-9}{7x+3}\right)$

$\dfrac{9-9}{-21+3} = \boxed{0}$

8. $\lim_{x\to -3}\left(\dfrac{x^2-9}{2x^2+7x}\right)$

$= \dfrac{9-9}{2(-3)^2+7(-3)} = \boxed{0}$

9. $\lim_{x\to 9}\left(\dfrac{\sqrt{x}-3}{-9}\right)$

$\dfrac{3-3}{-9} = \boxed{0}$

10. $\lim_{h\to 0}\left(\dfrac{(1+h)^2-1^2}{-4h}\right)$

11. $\lim_{h\to 0}\left(\dfrac{(3+h)^2-3^2}{3h}\right)$

12. $\lim_{h\to 0}\left(\dfrac{(x+h)^2-x^2}{2h}\right)$

Find the following limits for the piecewise function: $f(x) = \begin{cases} \boxed{x+1,\ x<2} \\ \boxed{x^2-2,\ 2<x<4} \\ \boxed{\sqrt{x+5},\ x\geq 4} \end{cases}$

13. $\lim_{x\to 1^+} f(x)$

2

14. $\lim_{x\to 1^-} f(x)$

2

15. $\lim_{x\to 1} f(x)$

2

16. $f(1)$

2

17. $\lim_{x\to 2^+} f(x)$

2

18. $\lim_{x\to 2^-} f(x)$

3

19. $\lim_{x\to 2} f(x)$

~~2~~ DNE

20. $f(2)$

DNE

21. $\lim_{x\to 3^+} f(x)$

7

22. $\lim_{x\to 3^-} f(x)$

7

23. $\lim_{x\to 3} f(x)$

7

24. $f(3)$

7

25. $\lim_{x\to 4^+} f(x)$

3

26. $\lim_{x\to 4^-} f(x)$

14

27. $\lim_{x\to 4} f(x)$

DNE

28. $f(4)$

3