

Use the definition of the derivative to answer problems 1-2.

1. For each function $f(x)$ given below, find the equation of the tangent line at the indicated point.

(a) $f(x) = x - x^2$ at $(2, -2)$

(b) $f(x) = 1 - 3x^2$ at $(0, 1)$

(c) $f(x) = \frac{1}{2x}$ at $x = 1$ $(1, \frac{1}{2})$

(d) $f(x) = x + \sqrt{x}$ at $x = 1$ $(1, 2)$

a. $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \frac{(2+h) - (2+h)^2 - (-2)}{h} = \frac{2+h - 4-4h-h^2 + 2}{h} = \frac{-h^2 - 3h}{h} = -h - 3 = -3$

$y + 2 = (x - 2)(-3)$

b. $\lim_{h \rightarrow 0} \frac{f(0+h) - f(0)}{h} = \frac{1 - 3(0+h)^2 - 1}{h} = \frac{-3h}{h} = -3$ inst. ROC

$y - 1 = -3(x - 0)$

c. $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \frac{\frac{1}{2(1+h)} - (\frac{1}{2}) \cdot \frac{1+h}{1+h}}{h} = \frac{\frac{1}{2(1+h)} - \frac{1}{2}}{h} = \frac{\frac{1 - (1+h)}{2(1+h)}}{h} = \frac{-h}{2(1+h)} = -\frac{1}{2}$ inst. ROC

$y - \frac{1}{2} = -\frac{1}{2}(x - 1)$

d. $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \frac{1+h + \sqrt{1+h} - 2}{h} = \frac{(-1+h) + \sqrt{1+h}}{h} \cdot \frac{(-1+h) - \sqrt{1+h}}{(-1+h) - \sqrt{1+h}} = \frac{(-1+h)^2 - (1+h)}{h((-1+h) - \sqrt{1+h})} = \frac{1 - 2h + h^2 - 1 - h}{h(-1+h - \sqrt{1+h})} = \frac{-3h + h^2}{h(-1+h - \sqrt{1+h})} = \frac{-3 + h}{-1+h - \sqrt{1+h}} = \frac{-3}{-2} = \frac{3}{2}$ inst. ROC

$y - 2 = \frac{3}{2}(x - 1)$

2. Given $f(x) = ax^2 + 2x$ and $f'(1) = 5$, solve for a .

$\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = 5 \rightarrow \frac{a(1+h)^2 + 2(1+h) - (a+2)}{h} = 5 \rightarrow \frac{a + 2ha + h^2a + 2 + 2h - a - 2}{h} = 5$

$\rightarrow \frac{2ha + h^2a + 2h}{h} = 5 \rightarrow \frac{h(2a + ha + 2)}{h} = 5 \rightarrow 2a + ha + 2 = 5 \rightarrow 2a + (0)a + 2 = 5$

$\rightarrow 2a + 2 = 5 \rightarrow 2a = 3 \rightarrow a = \frac{3}{2}$

3. An object falls to earth's surface and can be modeled by $g(x) = 4x^2$. What is the average speed for the time period starting at $x=2$ seconds and lasting 0.8 seconds?

interval: $[2, 2.8]$

$\frac{f(2.8) - f(2)}{2.8 - 2} = \frac{31.36 - 16}{2.8 - 2} = \frac{15.36}{0.8} = 19.2 \frac{\text{units}}{\text{seconds}}$