Determine if each are continuous everywhere. If yes use the 3 step method to prove. If no, state where discontinuous and why.

1. $f(x)=\begin{cases} x^2, & for \quad x>2 \\ 2x, & for \quad x\le 2 \end{cases}$

① $\lim_{x\to 2} f(x) = 4$

② $f(2) = 4$

③ yes - $\lim_{x\to 2} f(x) = f(2)$

2. $f(x)=\begin{cases} 2x+3, & for \quad x\ge 0 \\ -2x-1, & for \quad x<0 \end{cases}$

① $\lim_{x\to 0} f(x) = DNE$

② $f(0) = 3$

③ $\lim_{x\to 0} f(x) \ne f(0)$

NO

jump discont. at $x=0$

3. $f(x)=\begin{cases} x^2+3, & for \quad x>-1 \\ 2x+6, & for \quad x<-1 \\ 2, & for \quad x=-1 \end{cases}$

① $\lim_{x\to -1} f(x) = 4$

② $f(-1) = 2$

③ no $\lim_{x\to -1} f(x) \ne f(-1)$

~~jump~~ discont. removable at $x=-1$

4. $f(x)=\dfrac{x-1}{x^2+x-2} = \dfrac{\cancel{(x-1)}}{(x+2)\cancel{(x-1)}}$

discontinuities at

$x=-2$: VA (infinite)

$x=1$ (removable)

5. $f(x)=\begin{cases} x+1, & x\le 1 \\ 2x^2, & x>1 \end{cases}$

① $\lim_{x\to 1} f(x) = 2$

② $f(1) = 2$

③ yes - $\lim_{x\to 1} f(x) = f(1)$

6. $f(x)=\begin{cases} x^2+6x-1, & x\ne 3 \\ 2, & x=3 \end{cases}$

① $\lim_{x\to 3} f(x) = 26$

② $f(3) = 2$

③ no - $\lim_{x\to 3} f(x) \ne f(3)$

removable discont. at $x=3$

7. $f(x)=\begin{cases} 2x-5, & x<2 \\ \sqrt{x+1}, & x\ge 2 \end{cases}$

① $\lim_{x\to 2} f(x) = DNE$

② $f(2) = \sqrt{3}$

③ no - $\lim_{x\to 2} f(x) \ne f(2)$

jump discont. at $x=2$

8. $f(x)=\dfrac{3}{x+2}$

no

$x=-2$

infinite discont.

9. $f(x)=\dfrac{\cancel{(x+3)}(x+5)}{(x+2)\cancel{(x+3)}}$

$x=-3, -2$

removable infinite

10. $f(x)=\begin{cases} x^3, & for \quad x\ge 2 \\ 2x+3, & for \quad x<2 \end{cases}$

① $\lim_{x\to 2} f(x) = DNE$

② $f(2) = 8$

③ no $\Rightarrow \lim_{x\to 2} f(x) \ne f(2)$

jump at $x=2$